The problem of the initial conditions in flavoured leptogenesis and the tauon N_2-dominated scenario

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April 12, 2011

Abstract

We discuss the conditions to realize a scenario of 'strong thermal leptogenesis', where the final asymmetry is fully independent of the initial conditions, taking into account both heavy and light neutrino flavor effects. In particular, the contribution to the final asymmetry from an initial pre-existing asymmetry has to be negligible. We show that in the case of a hierarchical right-handed (RH) neutrino mass spectrum, the only possible way is a N_2-dominated leptogenesis scenario with a lightest RH neutrino mass $M_1 \ll 10^9\,\mathrm{GeV}$ and with a next-to-lightest RH neutrino mass $10^{12}\,\mathrm{GeV} \gg M_2 \gg 10^9\,\mathrm{GeV}$. This scenario necessarily requires the presence of a heaviest third RH neutrino specie. Moreover, we show that the final asymmetry has to be dominantly produced in the tauon flavour while the electron and the muon asymmetries have to be efficiently washed out by the lightest RH neutrino inverse processes. Intriguingly, such seemingly special conditions for successful strong thermal leptogenesis are naturally fulfilled within $SO(10)$-inspired models. Besides the tauon N_2-dominated scenario, successful strong thermal leptogenesis is also achieved in scenarios with quasi-degenerate RH neutrino masses. We also comment on the supersymmetric case. We also derive an expression for the final asymmetry produced from leptogenesis taking fully into account heavy neutrino flavour effects in the specific case $M_1 \gg 10^{12}\,\mathrm{GeV}$ (heavy flavoured scenario), a result that can be extended to any other mass pattern.

1 Introduction

With the discovery of neutrino masses and mixing in neutrino oscillations experiments, leptogenesis [1], a direct cosmological implication of the seesaw mechanism [2], can be regarded as the most attractive model of baryogenesis. When the minimal thermal scenario is considered, the predicted final asymmetry depends on 18 additional parameters introduced by a minimal type I seesaw with three RH neutrinos.

The low energy neutrino experiments can only test the nine parameters in the light neutrino mass matrix, whereas leptogenesis imposes just a condition on the baryon-to-photon number ratio [3]

$$\eta_B^{\text{CMB}} = (6.2 \pm 0.15) \times 10^{-10} \,, \tag{1}$$

so that testing the seesaw mechanism and leptogenesis, is seemingly impossible.

In order to circumvent this intrinsic obstacle, two complementary (but not mutually exclusive) strategies are usually considered. A first strategy is to restrict the parameter space imposing extra conditions coming from models of new physics. A remarkable example is provided by $SO(10)$ models inspired conditions. Imposing successful leptogenesis yields specific predictions on the low energy neutrino parameters [4, 5]. Recently, conditions required by leptogenesis within models of discrete flavour symmetries attracted great attention as well [6, 7]. A second strategy is to look for additional phenomenological constraints on the seesaw mechanism parameter space in addition to low energy neutrino experiments and leptogenesis. Additional constraints within models beyond the SM resulting for example from a determination of slepton masses in SUSY models, from lepton flavor violation processes, from electric dipole moments, from attempts to explain dark matter with RH neutrinos, from collider physics, have been extensively studied [8].

However, even when the large number of free parameters is restricted by some assumption or model of new physics or over-constrained by supplementary phenomenologies, there is one further legitimate conceptual issue to be addressed. The predicted final asymmetry could depend, in addition to the seesaw parameters, on the details of the cosmological history as well. These could for example determine the initial abundance of the heavy right-handed neutrinos, whose decays are responsible for the production of the asymmetry, and a possible non-vanishing value of the $B - L$ asymmetry before the onset of leptogenesis. In this respect it should be underlined that there are many ways how a large pre-existing $B - L$ asymmetry could have been generated in the latest stage of inflation, for example via Affleck-Dine mechanism [9], by gravity [10], or, after inflation and before the onset of leptogenesis, even by more traditional GUT bosons decays [11].

Apparently, this second obstacle can be more easily circumvented. After all, standard

Big Bang Nucleosynthesis provides already an example of successful calculation of few observable cosmological quantities (the primordial nuclear abundances) depending both on particle physics parameters and, potentially, on the details of the history of the early Universe. By assuming initial thermal conditions, for a reheating temperature after inflation higher than $\sim 1\,\mathrm{MeV}$, it is possible to calculate the primordial nuclear abundances independently of the initial conditions.

Encouraged by this relevant example, one can assume a thermal scenario of leptogenesis and calculate the final asymmetry almost independently of a detailed knowledge of the initial conditions and of many other possible cosmological intervening complex processes. The reheating temperature has then to be high enough that the Yukawa interactions alone can efficiently thermalize the right-handed neutrino abundance. However, even though this condition guarantees a thermal production of the asymmetry, it is still not sufficient to guarantee independence of the initial conditions. Throughout the paper, we will refer to a scenario of leptogenesis where the final asymmetry is independent of the initial conditions, as *strong thermal leptogenesis*. Moreover, we will always refer to the case of a hierarchical RH neutrino spectrum, commenting in Section 5 on the case of scenarios with quasi-degenerate RH neutrino masses.

A sufficient set of conditions for strong thermal leptogenesis usually involves additional constraints on the see-saw parameters further restricting the parameter space. More specifically, the independence of the initial conditions typically translates into a condition of strong wash-out regime. In this case any non-thermal or pre-existing contribution to the final asymmetry is unavoidably washed-out by the same processes that are already predicted by the see-saw mechanism (namely inverse processes involving RH neutrinos) without any need to impose new ad hoc conditions or to extend the minimal seesaw framework. The conditions for strong thermal leptogenesis are particularly simple when flavor effects are neglected. They basically reduce to a simple condition on the value of the decay parameter K_i of the RH neutrino N_i responsible for the generation of the asymmetry, $K_i \gtrsim K_\star \gg 1$ [12]. The precise value of $K_\star$ depends just on the value of the initial pre-existing asymmetry. For example, as we will see, for an initial very large pre-existing asymmetry $\mathcal{O}(1)$ one has $K_\star \simeq 10$, but it is in any case a constant value independent of the see-saw parameters.

On the other hand, when light neutrino flavor effects [13] are taken into account, the conditions become much more involved. An analysis of the conditions for the independence of the initial RH neutrino abundance shows that the constraints on the parameter space are even stronger [14]. They still result into a condition on the decay parameter $K_i \gtrsim K_\star$, where, however, now $K_\star$ is not just a constant value but it depends on the seesaw

parameters. Therefore, the precise condition for the independence of the initial conditions has to be determined at each point in the seesaw parameter space.

In [15] it was pointed out that there are no conditions able to guarantee an efficient wash-out of an arbitrary pre-existing asymmetry from the lightest RH neutrino inverse processes, unless these are lighter than $\sim 10^9\,\mathrm{GeV}$. On the other hand, a complete study of the conditions for the wash-out of a pre-existing asymmetry, accounting also for the washout from the heavier RH neutrinos (heavy neutrino flavour effects) and their compatibility with successful leptogenesis (an important point in our analysis) is still missing.

One could simply think that conditions similar to those found in [14] for the independence of the RH neutrino abundance should hold. However, in this paper, we will show that the conditions for the wash-out of a pre-existing asymmetry are more elaborate and even more stringent. They do not just involve the value of the decay parameters but even the RH neutrino mass spectrum and the flavour composition of the final asymmetry themselves. We will show, quite remarkably, that when the mass spectrum is assumed to be hierarchical, only one specific mass pattern can allow for successful strong thermal leptogenesis [1].

Moreover we will show that when heavy neutrino flavors are taken into account, because of new subtle effects, even the requirements for the independence of the initial RH neutrino abundance become more stringent. In the end we will show that the set of sufficient conditions to be imposed in order to have an independence of the initial asymmetries will be also sufficient for an independence of the initial RH neutrino abundances.

In the second section we discuss how the wash-out of an initial asymmetry can proceed in different ways depending on the specific RH neutrino mass pattern and therefore we show how one has to distinguish different cases. In the third section we start by studying the so called 'heavy flavored scenario', where the leptons can be treated as a coherent superposition of light (e, μ, τ) flavor eigenstates. In the fourth section we consider 'light flavored scenarios', concluding that only a pattern where the lightest RH neutrino wash-out occurs in the three-light flavored regime, for $M_1 \ll 10^9\,\mathrm{GeV}$, can guarantee a total wash-out of a pre-existing asymmetry. Furthermore we will show that if one also imposes successful leptogenesis, then necessarily one also has the additional condition $10^{12}\,\mathrm{GeV} \gg M_2 \gg 10^9\,\mathrm{GeV}$ and that the final asymmetry has to be dominantly produced by the

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[1]This conclusion is valid in non supersymmetric models. As we will comment in the last section, supersymmetric models offer an interesting loophole. Notice also that this conclusion does not translate into a prediction on the light neutrino mass spectrum, this would require more constrained models where the heavy neutrino and the light neutrino masses are linked.

next-to-lightest RH neutrinos N_2 (N_2-dominated scenario [16]) in the tauon flavour. In section 5 we make some final remarks (e.g. on supersymmetric models), we discuss some caveats, and draw the conclusions.

2   Heavy and light neutrino flavors

The seesaw mechanism relies on the addition to the SM lagrangian of RH neutrinos with Yukawa couplings and a Majorana mass term,

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + i\overline{N_{Ri}}\gamma_\mu\partial^\mu N_{Ri} - h_{\alpha i}\overline{\ell_{L\alpha}}N_{Ri}\tilde{\Phi} - \frac{1}{2}M_i\overline{N_{Ri}^c}N_{Ri} + h.c. \quad (i = 1, 2, 3, \quad \alpha = e, \mu, \tau). \tag{2}$$

For definiteness and simplicity we will consider the case of three RH neutrinos species, though all data from low energy neutrino experiments can be explained within a more minimal 2 RH neutrino model. Interestingly, we will see that the conditions of strong thermal leptogenesis will require in the end the presence of a third RH neutrino specie.

After spontaneous symmetry breaking, a Dirac mass term $m_D = v\, h$, is generated by the vev $v = 174$ GeV of the Higgs boson. In the see-saw limit, $M \gg m_D$, the spectrum of neutrino mass eigenstates splits in two sets: 3 very heavy neutrinos, N_1, N_2 and N_3 respectively with masses $M_1 \leq M_2 \leq M_3$ and almost coinciding with the eigenvalues of M, and 3 light neutrinos with masses $m_1 \leq m_2 \leq m_3$, the eigenvalues of the light neutrino mass matrix given by the see-saw formula [2],

$$m_\nu = -m_D\,\frac{1}{M}\,m_D^T\,. \tag{3}$$

Neutrino oscillation experiments measure two neutrino mass-squared differences. For normal schemes one has $m_3^2 - m_2^2 = \Delta m_{\text{atm}}^2$ and $m_2^2 - m_1^2 = \Delta m_{\text{sol}}^2$, whereas for inverted schemes one has $m_3^2 - m_2^2 = \Delta m_{\text{sol}}^2$ and $m_2^2 - m_1^2 = \Delta m_{\text{atm}}^2$. For $m_1 \gg m_{\text{atm}} \equiv \sqrt{\Delta m_{\text{atm}}^2 + \Delta m_{\text{sol}}^2} = (0.050 \pm 0.001)\,\text{eV}$ [17] the spectrum is quasi-degenerate, while for $m_1 \ll m_{\text{sol}} \equiv \sqrt{\Delta m_{\text{sol}}^2} = (0.0087 \pm 0.0001)\,\text{eV}$ [17] it is fully hierarchical (normal or inverted). The most stringent upper bound on the absolute neutrino mass scale is derived from cosmological observations. Recently, quite a conservative upper bound,

$$m_1 < 0.19\,\text{eV} \quad (95\%\,\text{CL})\,, \tag{4}$$

has been obtained by the WMAP collaboration combining CMB, baryon acoustic oscillations and the Hubble Space Telescope measurement of H_0 [3].

With leptogenesis, this simple extension of the Standard Model is also able to explain the observed baryon asymmetry of the Universe eq. (1). This is generated by the *CP*

violating decays of the RH neutrinos into leptons, $N_i \to \ell_i + H^\dagger$, and into anti-leptons, $N_i \to \bar{\ell}_i + H$, producing a lepton asymmetry that for temperatures $T \gtrsim 100\,\mathrm{GeV}$ is partially converted into a baryon asymmetry by sphaleron $(B-L$ conserving) processes. The observed baryon asymmetry can then be calculated from the final $B-L$ asymmetry simply using

$$\eta_B = a_{\mathrm{sph}} \frac{N^{\mathrm{f}}_{B-L}}{N^{\mathrm{rec}}_\gamma} \simeq 0.96 \times 10^{-2}\, N^{\mathrm{f}}_{B-L}\,, \tag{5}$$

where we indicate with N_X any particle number or asymmetry X calculated in a portion of co-moving volume containing one heavy neutrino in ultra-relativistic thermal equilibrium, i.e. such that $N^{\mathrm{eq}}_{N_2}(T \gg M_2) = 1$.

The leptons produced in N_i decays can be described in terms of quantum states that we indicate with $|\ell_i\rangle$. They have a flavor composition given by

$$|\ell_i\rangle = \sum_\alpha \mathcal{C}_{i\alpha}\, |\ell_\alpha\rangle\,, \quad \mathcal{C}_{i\alpha} \equiv \langle \ell_\alpha|\ell_i\rangle \quad (\alpha = e, \mu, \tau) \tag{6}$$

that is in general distinct for each heavy neutrino flavour i. We will refer to them as 'heavy neutrino flavours' lepton quantum states and analogously we will refer to the $|\ell_\alpha\rangle$ as the 'light neutrino flavours' lepton quantum states. Similarly, one can write for the anti-leptons

$$|\bar{\ell}'_i\rangle = \sum_\alpha \bar{\mathcal{C}}_{i\alpha}\, |\bar{\ell}_\alpha\rangle\,, \quad \bar{\mathcal{C}}_{i\alpha} \equiv \langle \bar{\ell}_\alpha|\bar{\ell}'_i\rangle\,, \quad (\alpha = e, \mu, \tau)\,. \tag{7}$$

Notice that while the light neutrino flavours quantum states form an orthonormal basis,

$$\langle \ell_\alpha|\ell_\beta\rangle = \delta_{\alpha\beta} \quad \text{and} \quad \langle \bar{\ell}_\alpha|\bar{\ell}_\beta\rangle = \delta_{\alpha\beta}\,, \tag{8}$$

in general the heavy neutrino flavour quantum states do not [18], since in general the quantities $\langle \ell_j|\ell_i\rangle$ do not vanish. In the Appendix we give an expression for the $p_{ij} \equiv |\langle \ell_j|\ell_i\rangle|^2$ in terms of the Dirac mass matrix showing this point. Analogously, the pre-existing lepton quantum states $|\ell_{\mathrm{p}}\rangle$ and anti-lepton quantum states $|\bar{\ell}'_{\mathrm{p}}\rangle$ also have a flavour composition, given respectively by

$$|\ell^{\mathrm{p}}\rangle = \sum_\alpha \mathcal{C}_{\mathrm{p}\alpha}\, |\ell_\alpha\rangle\,, \quad \mathcal{C}_{\mathrm{p}\alpha} \equiv \langle \ell_\alpha|\ell^{\mathrm{p}}\rangle \quad (\alpha = e, \mu, \tau) \tag{9}$$

and

$$|\bar{\ell}^{\mathrm{p}}{}'\rangle = \sum_\alpha \bar{\mathcal{C}}_{\mathrm{p}\alpha}\, |\bar{\ell}_\alpha\rangle\,, \quad \bar{\mathcal{C}}_{\mathrm{p}\alpha} \equiv \langle \bar{\ell}_\alpha|\bar{\ell}^{\mathrm{p}}{}'\rangle \quad (\alpha = e, \mu, \tau)\,. \tag{10}$$

These leptons are those ones that, in the standard cosmological picture, would be produced at the end or soon after inflation, before the RH neutrino processes become effective.

In general the flavour composition of leptons and of CP conjugated anti-leptons are different. However, we will first derive our conclusions assuming that they are equal, i.e. $\mathcal{C}_{p\alpha} = \bar{\mathcal{C}}_{p\alpha}$ and then, in section 5, we will point out that even allowing for a different flavour composition of the pre-existing leptons and anti-leptons our conclusions do not change.

While flavor blind gauge interactions preserve the coherence of the lepton quantum states, Yukawa charged lepton interaction are flavor sensitive and act as a potential source of decoherence [18, 19] which can be neglected only for sufficiently high values of the decaying RH neutrino masses [20]

$$M_i \gg 10^{12}\,\mathrm{GeV}\,. \tag{11}$$

On the other hand, if

$$10^{12}\,\mathrm{GeV} \gg M_i \gg 10^9\,\mathrm{GeV}\,, \tag{12}$$

the tauon charged lepton interactions are on average fast enough to destroy the coherent evolution of the quantum superposition of the tauon and of the its orthogonal component (superposition of the electron and of the muon component) before the lepton quantum state interacts inversely with an Higgs boson to produce a RH neutrino N_i. Therefore, in this two light flavor regime, the lepton quantum states inverse decay as an incoherent mixture of tauon flavor eigenstates plus the (still) coherent τ orthogonal superposition of the electron and of the muon components. Therefore, the wash-out from inverse decays has to be taken into account separately on the asymmetry in the tauon flavour and on the asymmetry in the tauon orthogonal flavour component that we will indicate with $\tilde{\tau}$ [2].

Finally, if

$$M_i \ll 10^9\,\mathrm{GeV}\,, \tag{13}$$

then even the coherence of the superposition of the electron and of the muon component breaks down and the lepton quantum states, at $T \sim M_i$, have to be described as a fully incoherent mixture of three light flavor eigenstates (three-flavor regime).

In our discussion we will assume that the three RH neutrino masses satisfy one of the previous conditions (eq. (11), eq. (12) or eq. (13)), in such a way that the collisions of the lepton doublets with the charged leptons are fast enough that a classical Boltzmann description provides a good approximation [21, 22, 23]. We will also assume a hierarchical RH neutrino mass spectrum with $M_{i+1} \gtrsim 3\,M_i$ in a way that decays and wash-out for each RH neutrino specie occur in separate stages that do ont overlap with each other [25].

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[2]In general, throughout the paper, given a flavour x, we will indicate with $\tilde{x}$ the set of the orthogonal flavours corresponding to the plane orthogonal to x in flavour space.

There is another source of decoherence of the leptonic quantum states $|\ell_i\rangle$ produced from N_i RH neutrino decays and on the the $|\ell^{\mathrm{p}}\rangle$. Notice that in general the $|\ell_i\rangle$ do not form an orthonormal basis (see Appendix), meaning that in general $\langle \ell_i | \ell_{j\neq i} \rangle \neq 0$. At the same time the $|\ell^{\mathrm{p}}\rangle$ have an arbitrary flavour composition (e.g. an equal mixture of electron, muon and tauon components). This implies that the quantum lepton states $|\ell_i\rangle$ and $|\ell_{\mathrm{p}}\rangle$, after an inverse process producing a RH neutrino $N_{j\neq i}$, become an incoherent mixture of a $|\ell_j\rangle$ component and of a $|\ell_j\rangle$ orthogonal component that, in our convention, it will be indicated by $|\ell_{\tilde{j}_i}\rangle$ [24, 15]. Also in this case, we will employ a classical Boltzmann description for the evolution of the asymmetries, assuming that at each inverse process, the $|\ell_i\rangle$ (or the $|\ell_{\mathrm{p}}\rangle$) quantum state instantaneously collapses either into the $|\ell_j\rangle$ parallel component or into its orthogonal one that we will indicate with $|\ell_{\tilde{j}_i}\rangle$ (or $|\ell_{\tilde{j}_{\mathrm{p}}}\rangle$). Again the classical picture holds when the interactions are fast enough [21, 22, 23], in this case with respect to the expansion rate since there are no other involved active processes [3].

The potential simultaneous interplay of six different flavors of leptons, the three heavy ones and three light ones, the non-orthogonality of the three heavy flavors in contrast with the orthogonality of the three light flavors, yield different scenarios corresponding to different RH neutrino mass patterns that need to be discussed separately. In the next sections we will discuss each case separately but it is useful to make before some general comments.

Because of the linearity of the Boltzmann equations, the final $B-L$ asymmetry is the sum of two contributions,

$$N_{B-L}^{\mathrm{f}} = N_{B-L}^{\mathrm{p,f}} + N_{B-L}^{\mathrm{lep,f}} \,. \tag{14}$$

The first term is the residual value of a pre-existing asymmetry, after the RH neutrinos wash-out, whereas the second term is the final value of the $B-L$ asymmetry produced by the RH neutrino decays, the product of leptogenesis genuinely depending only on the seesaw parameters.

The main goal of this paper is to show under which conditions one can have simultaneously both successful leptogenesis and a negligible contribution from the pre-existing

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[3]Within a full quantum density matrix formalism, corrections to the classical picture are expected to be maximal in an intermediate regime of mild interaction rates. However, they cannot be particularly relevant in the case of hierarchical RH neutrino mass patterns. This is because the N_j inverse processes occur when the the $|\ell_i\rangle$ production from the N_i decays has already switched off and therefore, there is no overlap between the N_i decays and the N_j inverse decays. In this situation the damping of the density matrix off-diagonal terms is rather fast. As we will see our conclusions will not rely on a detailed kinetic description. On the other hand a density matrix treatment seems to be unavoidable if one considers RH neutrino mass patterns beyond the hierarchical limit, when decays and inverse processes involving different RH neutrino species occur simultaneously and compete with each other.

asymmetry, i.e.

$$|N^{\mathrm{p,f}}_{B-L}| \ll |N^{\mathrm{lep,f}}_{B-L}|\,. \tag{15}$$

Only in this way it is guaranteed that the successful leptogenesis condition, $\eta^{\mathrm{f}}_B = \eta^{CMB}_B$, really constraints the seesaw parameter space. Notice that we cannot completely ignore $N^{\mathrm{lep,f}}_{B-L}$, since, after having found the conditions for (15) to hold, we have also to check that these do not prevent successful leptogenesis. As we will see, this check will not require an explicit calculation of $N^{\mathrm{lep,f}}_{B-L}$ and we will mainly focus on the evolution of $N^{\mathrm{p,f}}_{B-L}$. However, in the Appendix, we also explicitly derive the solution for $N^{\mathrm{lep,f}}_{B-L}$ for a specific RH neutrino mass pattern, in the so called 'heavy flavoured scenario'.

Notice that we will neglect throughout sections 3 and 4 a new effect studied in [27], so called light neutrino flavour coupling. However, we will comment in section 5 about the kind of impact that light neutrino flavour coupling might have on our conclusions.

It is also useful to make a general remark about the flavour composition of the pre-existing leptons and anti-leptons. The asymmetry $N^{\mathrm{p,i}}_{B-L}$ is distributed not only in the lepton doublets but also in the RH charged leptons and in the quarks. We will always assume that $T_i \lesssim 10^{14}\,\mathrm{GeV}$ so that sphaleron processes are in equilibrium and the asymmetry distribution among the different particle species can be calculated from equilibrium conditions [26]. In particular the asymmetry in the lepton doublets is related to the $B-L$ asymmetry by

$$N_L = \alpha_L\, N_{B-L}\,, \tag{16}$$

with $\alpha_L \simeq -2/3$. Assuming that only the RH neutrino decays and inverse processes can modify the $B-L$ asymmetry, any change of N_{B-L} can be only triggered by a modification of N_L, given by the difference of the number of leptons and anti-leptons.

On practical grounds, our objective is to calculate the final value of the wash-out factor of a pre-existing asymmetry defined as

$$w^{\mathrm{f}} \equiv \frac{N^{\mathrm{p,f}}_{B-L}}{N^{\mathrm{p,i}}_{B-L}}\,. \tag{17}$$

Conservatively, for a pre-existing asymmetry $N^{\mathrm{p,i}}_{B-L} = \mathcal{O}(1)$, e.g. generated by an Affleck-Dine mechanism during inflation [9], the independence of the initial conditions requires $w^{\mathrm{f}} \ll 10^{-8}$.

So far, this wash-out factor has been calculated only in a completely unflavoured case [12], i.e. neglecting both light and heavy neutrino flavour effects. It is useful, also in order to set up the notation, to review briefly this simple calculation.

In this simple picture, the pre-existing asymmetry has no flavour composition. Assuming $M_3, M_2 \gg T_i \gg M_1$, one can just consider the wash-out from the lightest RH

neutrinos N_1. The $B - L$ asymmetry is then described by a particularly simple set of just two Boltzmann equations [28],

$$\frac{dN_{N_1}}{dz_1} = -D_1 \left(N_{N_1} - N_{N_1}^{\text{eq}} \right),$$

(18)

$$\frac{dN_{B-L}}{dz_1} = \varepsilon_1 D_1 \left(N_{N_1} - N_{N_1}^{\text{eq}} \right) - N_{B-L} W_1(z_1),$$

(19)

where we defined $z_i \equiv M_i/T$ $(i = 1, 2, 3)$. The expansion rate is expressed as

$$H = \sqrt{\frac{8\,\pi^3\,g_\star}{90}} \frac{M_i^2}{M_{\text{Pl}}} \frac{1}{z_i^2} \simeq 1.66\,\sqrt{g_\star}\,\frac{M_i^2}{M_{\text{Pl}}} \frac{1}{z_i^2},$$

(20)

having indicated with $g_\star = g_{SM} = 106.75$ the total number of degrees of freedom and with $M_{\text{Pl}} = 1.22 \times 10^{19}\,\text{GeV}$ the Planck mass.

If we indicate with Γ_i the decay rate of the RH neutrinos N_i into leptons, and with $\bar{\Gamma}_i$ the decay rate into anti-leptons, the total decay rate, $\Gamma_{\text{D},i} \equiv \Gamma_i + \bar{\Gamma}_i = \widetilde{\Gamma}_i \langle 1/\gamma \rangle$, is the product of the decay width times the thermally averaged dilation factor, that can be expressed in terms of the ratio of the modified Bessel functions, $\langle 1/\gamma \rangle = \mathcal{K}_1(z)/\mathcal{K}_2(z)$.

The decay parameters are then defined as $K_i \equiv \widetilde{\Gamma}_i/H(z_i = 1) = \widetilde{m}_i/m_\star$, and can be expressed in terms the effective neutrino masses [29] and of the equilibrium neutrino mass [30, 12], respectively given by

$$\widetilde{m}_i = v^2 \frac{(m_D^\dagger m_D)_{ii}}{M_i} \quad \text{and} \quad m_\star = \frac{16\pi^{5/2}\sqrt{g_\star}}{3\sqrt{5}} \frac{v^2}{M_{Pl}} \simeq 1.08 \times 10^{-3}\,\text{eV}.$$

(21)

The decay terms and the related wash-out terms are given by

$$D_i \equiv \frac{\Gamma_{\text{D},i}}{H z} = K_i z_i \left\langle \frac{1}{\gamma_i} \right\rangle, \qquad W_i(z_i) = \frac{1}{2} D_i(z_i) N_{N_i}^{\text{eq}}(z_i) = \frac{1}{4} K_i \mathcal{K}_1(z_i) z_i^3,$$

(22)

where the equilibrium abundance has been also expressed in terms of the modified Bessel functions. Defining the total CP asymmetries as

$$\varepsilon_i \equiv -\frac{\Gamma_i - \bar{\Gamma}_i}{\Gamma_i + \bar{\Gamma}_i},$$

(23)

the final asymmetry is then simply given by

$$N_{B-L}^{\text{f}} = N_{B-L}^{\text{p,i}} e^{-\frac{3\pi}{8} K_1} + \varepsilon_1 \kappa(K_1),$$

(24)

where $\kappa(K_1)$ is the efficiency factor. Therefore, in this case one has $N_{B-L}^{\text{lep}} = \varepsilon_1 \kappa(K_1)$ while the wash-out factor of the pre-existing asymmetry is simply given by $w^{\text{f}} = e^{-3\pi K_1/8}$.

It is then sufficient to impose $K_1 \gtrsim 10$ in order to have $w^{\mathrm{f}} \lesssim 10^{-8}$, ensuring the wash-out of a $\mathcal{O}(1)$ pre-existing asymmetry [4]. Notice moreover that if $T_i \gtrsim M_2$, then any asymmetry produced from the next-to-lightest RH neutrinos is washed out by the N_1 inverse processes in the same way as the pre-existing asymmetry.

When flavour effects are taken into account, these conclusions change drastically. In [15] it was shown that a simple condition $K_1 \gtrsim 10$ is not sufficient to guarantee the complete wash-out of a $\mathcal{O}(1)$ pre-existing asymmetry. This is possible only if $M_1 \ll 10^9\,\mathrm{GeV}$ and if $K_{1\alpha} \equiv p_{1\alpha}\,K_1 \gtrsim 10$ for all $\alpha = e, \mu, \tau$, where we defined $p_{i\alpha} \equiv |\langle \ell_\alpha | \ell_i \rangle|^2$. However, such a drastic condition is not compatible with successful leptogenesis since any asymmetry produced from the heavier RH neutrinos is washed-out together with the pre-existing asymmetry as well while the lightest RH neutrino CP asymmetries are too low for an asymmetry to be generated from N_1 decays after the freeze out of wash-out processes.

We have therefore to extend the analysis to a more general case where the assumption $M_3, M_2 \gg T_i \gg M_1$ is relaxed, pinning down the conditions for successful strong thermal leptogenesis. The discussion has to be specialized considering all possible mass patterns characterized by a different interplay between light and heavy neutrino flavour effects. We start discussing the so called 'heavy flavored scenario', where all the three RH neutrino masses satisfy the condition eq. (11).

3   Heavy flavored scenario

In this scenario all three RH neutrino masses are much heavier than about $10^{12}\,\mathrm{GeV}$ (see Fig. 1). In this case the $|\ell_i\rangle$'s evolve coherently since the charged lepton interactions are ineffective in measuring their light neutrino flavor composition. We also assume $T_i \gg M_3$ so that all three RH neutrinos can wash-out the pre-existing asymmetry. If we prove that some fraction of the pre-existing asymmetry survives in this most conservative case, then some (at least equal) fraction necessarily survives if $T_i \ll M_3$ as well.

3.1   First stage: $T_i > T \gg M_3$

There are different stages in the evolution of $N_{B-L}^{\mathrm{p}}(z)$. In a first stage, for $T_i > T \gg M_3$, all RH neutrino processes are ineffective and the $B-L$ asymmetry remains constant. The flavour composition of the pre-existing lepton doublets quantum states can be regarded

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[4]For definiteness, throughout the paper we will also refer to the wash-out of a $\mathcal{O}(1)$ pre-existing asymmetry.
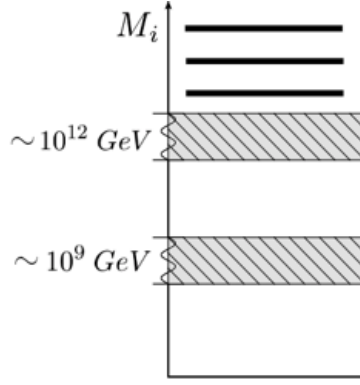
Figure 1: The mass pattern corresponding to the heavy flavoured scenario.

as a coherent superposition of a ℓ_3 parallel component and of a ℓ_3 orthogonal component, explicitly [5]

$$|\ell^p\rangle = \mathcal{C}_{p3}|\ell_3\rangle + \mathcal{C}_{p\tilde{3}}|\ell_{\tilde{3}}^{\mathrm{p}}\rangle\,, \quad \text{with} \quad p_{p3} + p_{p\tilde{3}} = 1\,, \tag{25}$$

where $p_{p3} \equiv |\mathcal{C}_{p3}|^2$ and $p_{p\tilde{3}} \equiv |\mathcal{C}_{p\tilde{3}}|^2$. This decomposition is pictorially represented in the upper-right panel of Fig. 2. The same decomposition can be made for the pre-existing anti-leptons and under the given assumptions one has $\bar{\mathcal{C}}_{p3} = \mathcal{C}_{p3}$ and $\bar{\mathcal{C}}_{p\tilde{3}} = \mathcal{C}_{p\tilde{3}}$. Correspondingly the pre-existing $B - L$ asymmetry can be also decomposed as

$$N_{B-L}^{\mathrm{p,i}} = N_{\Delta_3}^{\mathrm{p,i}} + N_{\Delta_{\tilde{3}}}^{\mathrm{p,i}}\,, \tag{26}$$

where we defined $N_{\Delta_3}^{\mathrm{p,i}} = p_{p3}\, N_{B-L}^{\mathrm{p,i}}$ and $N_{\Delta_{\tilde{3}}}^{\mathrm{p,i}} = (1 - p_{p3})\, N_{B-L}^{\mathrm{p,i}}$.

3.2   Second stage: $M_3 \gtrsim T \gtrsim T_{B3}$

Let us now discuss a second stage for $M_3 \gtrsim T \gtrsim T_{B3}$, where $T_{B3} \simeq M_3/z_{B3}$ is the freeze-out temperature of the N_3 inverse processes and $z_{B3} \simeq 2 + 4\, K_3^{0.13}\, e^{-2.5/K_3} = \mathcal{O}(1-10)$ [25]. An interaction of a quantum lepton state $|\ell^p\rangle$ with a Higgs boson can be regarded, in a classical statistical picture, as a measurement process where there is a probability p_{p3} that $|\ell^p\rangle$ is measured as a ℓ_3 producing a N_3 in the inverse decay and a probability $1 - p_{p3}$ that is measured as a $\ell_{\tilde{3}}$ and in this case no inverse process occurs [6]. In this way only the

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[5]Notice that with the notation $\ell_{\tilde{3}}$ we mean the projection of $|\ell^p\rangle$ on the plane orthogonal to $|\ell_3\rangle$ so that $\langle \ell_3 | \ell_{\tilde{3}} \rangle = 0$. More precisely we should write $\ell_{\tilde{3}_\mathrm{p}}$ but we imply the subscript p in order to simplify the notation. We will do the same for the projections on the planes orthogonal to $|\ell_2\rangle$ and $|\ell_1\rangle$.

[6]This is analogous to what happens in active-sterile neutrino oscillations described in terms classical Boltzmann equations [31, 23], where the orthogonal component here plays the role of the sterile component.
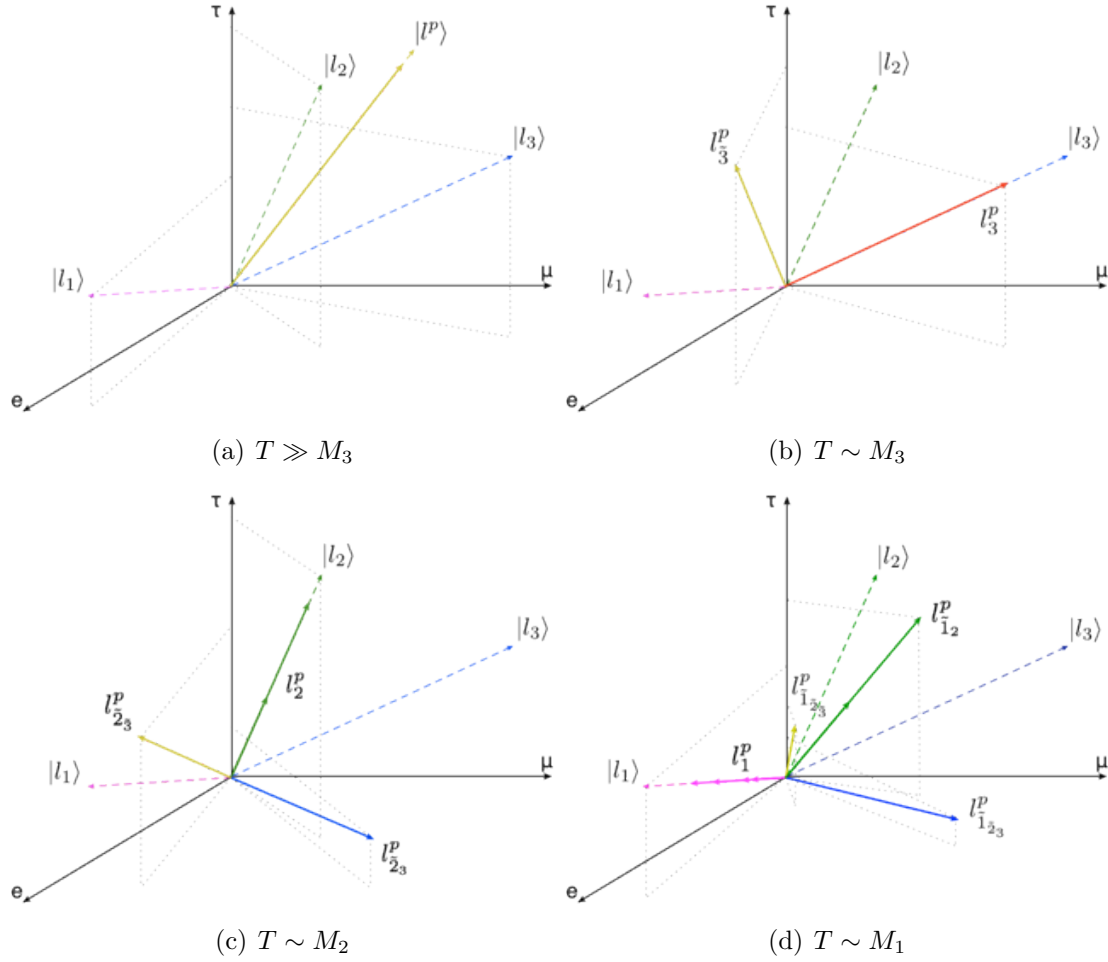
Figure 2: The four stages of the heavy flavoured scenario. In a first stage at $T \gg M_3$ (a) the pre-existing leptons are a coherent superposition of light flavor eigenstates. At $T \sim M_3$ (b) the N_3 decays and inverse processes break the coherent evolution of $|\ell_{\rm p}\rangle$ that becomes an incoherent mixture of a $|\ell_3\rangle$ component and of a $|\ell_{\bar{3}}\rangle$ component. At $T \sim M_2$ (c), the $|\ell_3\rangle$ and the $|\ell_{\bar{3}}\rangle$ components are both re-projected on a $|\ell_2\rangle$ component and on a $|\ell_{\bar{2}}\rangle$ component. We indicate with $\ell_i^{\rm P}$ or $\ell_{\tilde{i}_j}^{\rm P}$, those components of pre-existing leptons with a certain flavour composition, i and $\tilde{i}_j$, that give a contribution to the residual pre-existing asymmetry and that experienced a different wash-out history. For example $\ell_i^{\rm P}$ is the component of $\ell^{\rm P}$ that experienced only a wash-out from N_i RH neutrinos. On the other hand for example $\ell_{\tilde{i}_2}^{\rm P}$ is the component of $\ell^{\rm P}$ that has been first projected along ℓ_2, undergoing the wash-out from the N_2's, and then along a direction orthogonal to ℓ_1, escaping the wash-out from the N_1's. Notice that at a given stage components with a different wash-out history might end up in the same quantum state. At each stage the red components are those undergoing wash-out while the yellow ones are those that are unwashed. At $T \sim M_1$ (d), the $|\ell_2\rangle$ and the $|\ell_{\bar{2}}\rangle$ components are finally projected on a $|\ell_1\rangle$ and on a $|\ell_{\bar{1}}\rangle$ component. Again the different arrows just simply track the different histories. At this stage there are 8 contributions to the final asymmetry that experienced a different wash-out. The yellow component is completely unwashed.

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component $N_{\Delta_3}^{\mathrm{p,i}}$ of the $B - L$ asymmetry is washed out, while the orthogonal component $N_{\Delta_{\bar{3}}}^{\mathrm{p,i}}$ is basically untouched (see again the upper right panel in Figure 2) [7].

We also have to take into account the production of the ℓ_3's and of the $\bar{\ell}'_3$'s from the CP violating decays of the heaviest RH neutrinos N_3. Within the adopted classical statistical picture, an interaction acts as a measurement process on the quantum state that collapses in one of the two components. Therefore, we can use Boltzmann equations to describe the evolution of the asymmetry. These can be simply written as

$$\frac{dN_{N_3}}{dz_3} = -D_3 \left(N_{N_3} - N_{N_3}^{\mathrm{eq}} \right) , \tag{27}$$

$$\frac{dN_{\Delta_3}}{dz_3} = \varepsilon_3 \, D_3 \left(N_{N_3} - N_{N_3}^{\mathrm{eq}} \right) - W_3 \, N_{\Delta_3} , \tag{28}$$

$$\frac{dN_{\Delta_{\bar{3}}}}{dz_3} = 0 . \tag{29}$$

As we said, we give in the Appendix the calculation of the asymmetry produced from the decays, focusing here only on the evolution of the residual pre-existing asymmetry. The residual pre-existing asymmetry after this stage, at $T \sim T_{B3}$, will be given then by

$$N_{B-L}^{\mathrm{p}}(T_{B3}) = N_{\Delta_3}^{\mathrm{p,i}} \, e^{-\frac{3\pi}{8} K_3} + N_{\Delta_{\bar{3}}}^{\mathrm{p,i}} \tag{30}$$

$$= p_{p3} \, N_{B-L}^{\mathrm{p,i}} \, e^{-\frac{3\pi}{8} K_3} + (1 - p_{p3}) \, N_{B-L}^{\mathrm{p,i}} , \tag{31}$$

corresponding to a wash-out factor

$$w(T_{B3}) = p_{p3} \, e^{-\frac{3\pi}{8} K_3} + 1 - p_{p3} . \tag{32}$$

As one can see, there is no condition that can be imposed on the see-saw parameters able to guarantee an efficient wash-out of a pre-existing asymmetry with a generic flavour composition ($p_{p3} \neq 1$). If we impose $K_3 \gg 1$, at the end of the stage at $T \sim T_{B3}$, the lepton doublets are an incoherent mixture of ℓ_3 and $\ell_{\bar{3}}$ and analogously the anti-leptons are an incoherent mixture of $\bar{\ell}_3$ and $\bar{\ell}_{\bar{3}}$. The asymmetry in the ℓ_3 component is efficiently washed out (the first term in the RH side of the previous equation) but the asymmetry in the orthogonal component survives.

---

[7]Notice that we are neglecting what can be called 'heavy neurino flavour coupling' [15] due to spectator processes, mainly to the Higgs asymmetry. When this is taken into account the evolution of the two components $N_{\Delta_3}^{\mathrm{p,i}}$ and $N_{\Delta_{\bar{3}}}^{\mathrm{p,i}}$ is not completely independent of each other but couple to some extent. The result is that $N_{\Delta_{\bar{3}}}^{\mathrm{p,i}}$ is not completely washed-out while the component $N_{\Delta_3}^{\mathrm{p,i}}$ partly is. However, an account of this effect does not change our conclusions, as we point out in Section 5.

3.3 Third stage: $T_{B3} \gtrsim T \gtrsim T_{B2} \sim M_2/z_{B2}$

Let us then consider the subsequent third stage, for $T_{B3} \gtrsim T \gtrsim T_{B2} \sim M_2/z_{B2}$. At $T \sim M_2$ the N_2 inverse processes start to collapse the $|\ell_3\rangle$ and the $|\ell_{\bar{3}}\rangle$ either as a $|\ell_2\rangle$ or as the orthogonal component $|\ell_{\bar{2}}\rangle$. We can therefore repeat exactly the same decomposition as in the previous stage and write the residual pre-existing asymmetry at the end of the previous stage as the sum of two terms,

$$N^{\rm p}_{B-L}(T_{B3}) = N^{\rm p}_{\Delta_2}(T_{B3}) + N^{\rm p}_{\Delta_{\bar{2}}}(T_{B3}) \,, \tag{33}$$

where

$$
\begin{aligned}
N^{\rm p}_{\Delta_2}(T_{B3}) &= p_{32}\, N^{\rm p}_{\Delta_3}(T_{B3}) + p_{\bar{3}2}\, N^{\rm p}_{\Delta_{\bar{3}}}(T_{B3}) \tag{34}\\
&= p_{32}\, p_{p3}\, N^{\rm p,i}_{B-L}\, e^{-\frac{3\pi}{8}\,K_3} + p_{\bar{3}2}\,(1 - p_{p3})\, N^{\rm p,i}_{B-L} \,, \tag{35}
\end{aligned}
$$

and

$$
\begin{aligned}
N^{\rm p}_{\Delta_{\bar{2}}}(T_{B3}) &= (1 - p_{32})\, N^{\rm p}_{\Delta_3}(T_{B3}) + (1 - p_{\bar{3}2})\, N^{\rm p}_{\Delta_{\bar{3}}}(T_{B3}) \tag{36}\\
&= (1 - p_{32})\, p_{p3}\, N^{\rm p,i}_{B-L}\, e^{-\frac{3\pi}{8}\,K_3} + (1 - p_{\bar{3}2})\,(1 - p_{p3})\, N^{\rm p,i}_{B-L} \,. \tag{37}
\end{aligned}
$$

These are the two terms of the asymmetry that have to be used as initial condition at the beginning of the N_2 wash-out stage. The Boltzmann equations are obtained from the eqs. (39) written in the previous stage with the simple replacement of the label $3 \to 2$, explicitly

$$\frac{dN_{N_2}}{dz_2} = -D_2\,(N_{N_2} - N^{\rm eq}_{N_2}) \,, \tag{38}$$

$$\frac{dN_{\Delta_2}}{dz_2} = \varepsilon_2\, D_2\,(N_{N_2} - N^{\rm eq}_{N_2}) - W_2\, N_{\Delta_2} \,, \tag{39}$$

$$\frac{dN_{\Delta_{\bar{2}}}}{dz_2} = 0 \,. \tag{40}$$

We can therefore straightforwardly express the residual pre-existing asymmetry at $T \sim T_{B2}$ as [8]

$$N^{\rm p}_{B-L}(T_{B2}) = N^{\rm p}_{\Delta_2}(T_{B3})\, e^{-\frac{3\pi}{8}\,K_2} + N^{\rm p}_{\Delta_{\bar{2}\bar{3}}}(T_{B3}) \,. \tag{41}$$

Imposing $K_2, K_3 \gtrsim 10$, one can neglect all terms exponentially suppressed so that the wash-out factor reduces to

$$w(T_{B2}) \simeq (1 - p_{\bar{3}2})\,(1 - p_{p3}) \,. \tag{42}$$

---

[8]Notice that the notation $\ell^{\rm p}_{\bar{x}_{\bar{y}}}$ indicates a lepton with flavour obtained by projecting the pre-existing quantum lepton state $|\ell_{\rm p}\rangle$ first on a plane orthogonal to the flavour y and then to the plane orthogonal to the flavour x. Correspondingly $N^{\rm p}_{\Delta_{\bar{x}_{\bar{y}}}}$ indicates the asymmetry in that flavour.

This result shows that in general, even at this stage after the wash-out both from N_3 and from N_2 inverse processes, there is no condition that one can impose on the see-saw parameters able to guarantee an efficient wash-out of a generic pre-existing asymmetry.

3.4 Fourth stage: $T_{B2} \gtrsim T \gtrsim T_{B1} \sim M_1/z_{B1}$

The wash-out from the lightest RH neutrinos can now be straightforwardly calculated going along the same lines as in the previous stages. At the end of the N_1 wash-out, at $T \sim T_{B1}$, the asymmetry freezes at its final value and one has

$$N_{B-L}^{p}(T_{B1}) = N_{B-L}^{p,f} = N_{\Delta_1}^{p}(T_{B2}) \, e^{-\frac{3\pi}{8} K_1} + N_{\Delta_{\bar{1}}}^{p}(T_{B1}) \, . \tag{43}$$

The final residual value of the pre-existing asymmetry is the sum of 8 terms (they are explicitly written in the Appendix), where 7 of them undergo the wash-out exponential suppression either of just one (three terms), or of two (three terms) or of all three (1 term) RH neutrinos. There is clearly one component that escapes the wash-out of all three RH neutrinos. Imposing $K_1, K_2, K_3 \gtrsim 10$, all contributions that undergo the wash-out of at least one RH neutrino are suppressed below the observed value. The final wash-out factor will be then dominated only by the contribution coming from the completely unwashed term and it is therefore given by

$$w^{\mathrm{f}} \simeq (1 - p_{\bar{2}_{\bar{3}}1}) \, (1 - p_{\bar{3}2}) \, (1 - p_{p3}) \, . \tag{44}$$

It is clear that, barring very special situations, that should be realized with great precision, the wash-out of a pre-existing large asymmetry cannot be enforced. These special situations are realized either when the pre-existing leptons coincide with the ℓ_3 leptons ($p_{p3} = 1$) or when $|\ell_1\rangle, |\ell_2\rangle, |\ell_3\rangle$ form an orthonormal basis (in this case necessarily $p_{\bar{2}_{\bar{3}}1} = 1$).

The latter would correspond to special Dirac mass matrices corresponding a see-saw orthogonal matrix [32] that is either the identity or that is obtained from the identity permuting rows or columns. These special forms correspond to so called form dominance models [33] and are enforced typically by discrete flavour symmetries such as $A4$ [34] typically justified in order to reproduce tri-bimaximal mixing [35]. However, in the limit of exact form dominance (corresponding to the case of unbroken symmetry in flavour symmetry models) the total [16] and the flavoured CP asymmetries vanish [6, 36]. Therefore, a deviation from the orthonormality is necessary. For example, in models with flavour discrete symmetries, this deviation has to be of the order of the symmetry breaking parameter $\xi \sim 10^{-2}$ in order to generate the correct asymmetry. However, such small deviations would still yield $w^{\mathrm{f}} \sim \xi$, still not sufficiently small to guarantee an efficient wash-out of a large asymmetry $\mathcal{O}(1)$.

In the Appendix we also show the derivation of the contribution to the final asymmetry from the RH neutrino decays, the genuine contribution from leptogenesis. However, here we just give the result that should be regarded as a benchmark case that can be easily extended to all the other mass patterns.

The total final asymmetry is the sum of two terms

$$N_{B-L}^{\mathrm{lep}}(T_{B1}) = N_{\Delta_1}^{\mathrm{lep}}(T_{B1}) + N_{\Delta_{\bar{1}}}^{\mathrm{lep}}(T_{B1}) \,, \tag{45}$$

where

$$
\begin{aligned}
N_{\Delta_1}^{\mathrm{lep}}(T_{B1}) \;=\;& p_{21}\, p_{32}\, \varepsilon_3\, \kappa(K_3)\, e^{-\frac{3\pi}{8}\,(K_1+K_2)} \\
&+ p_{21}\, \varepsilon_2\, \kappa(K_2)\, e^{-\frac{3\pi}{8}\,K_1} \\
&+ p_{\bar{2}_31}\,(1-p_{32})\, \varepsilon_3\, \kappa(K_3)\, e^{-\frac{3\pi}{8}\,K_1} \\
&+ \varepsilon_1\, \kappa(K_1)
\end{aligned}
\tag{46}
$$

and

$$
\begin{aligned}
N_{\Delta_{\bar{1}}}^{\mathrm{lep}}(T_{B1}) \;=\;& (1-p_{21})\,[p_{32}\, \varepsilon_3\, \kappa(K_3)\, e^{-\frac{3\pi}{8}\,K_2} + \varepsilon_2\, \kappa(K_2)] \\
&+ (1-p_{\bar{2}_31})\,(1-p_{32})\, \varepsilon_3\, \kappa(K_3) \,.
\end{aligned}
\tag{47}
$$

The probabilities p_{ij}'s can be expressed as

$$p_{ij} = \frac{\left| (m_D^\dagger\, m_D)_{ij} \right|^2}{(m_D^\dagger\, m_D)_{ii}\, (m_D^\dagger\, m_D)_{jj}} \,. \tag{48}$$

This result is an example of how to take into account all flavour effects in a combined way and is another main result of our paper.

4   Light flavored scenarios

In this section we consider RH neutrino mass patterns where at least one M_i is below $10^{12}\,\mathrm{GeV}$. In this way the pre-existing lepton states, the $|\ell^{\mathrm{p}}\rangle$'s and the $|\ell_i\rangle$'s, get partially or fully (if $M_i \ll 10^9\,\mathrm{GeV}$) projected on the light neutrino flavour basis during the stages when the N_i decays and inverse processes are active.

4.1   Scenarios with τ flavour projection

We start considering first the the three mass patterns with $M_1 \gg 10^9\,\mathrm{GeV}$ (see Fig. 3), where only the τ component is 'measured' by the tauon Yukawa interactions.
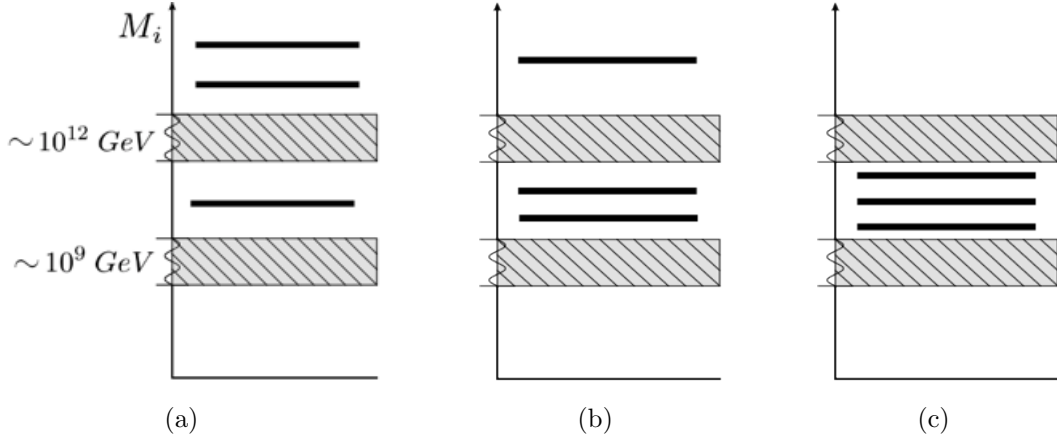
Figure 3: The three possible mass patterns where at least one RH neutrino mass is comprised between 10^9 GeV and 10^{12} GeV.

4.1.1 Case $M_{2,3} \gg 10^{12}$ GeV

Let us start with the case where the two heavier RH neutrino masses $M_{2,3} \gg 10^{12}$ GeV while 10^{12} GeV $\gg M_1 \gg 10^9$ GeV. The evolution of the residual pre-existing asymmetry $N_{B-L}^{\rm p}$ proceeds through the same steps discussed in the heavy flavoured scenario until the end of the N_2 washout at $T \sim T_{B2}$ when it is given by the eq. (41).

At this stage an important difference arises between the two scenarios. In the considered light flavoured scenario, before the onset of the N_1 washout processes, the tauon charged lepton interactions become effective. In this way the τ component of the quantum lepton states is measured and they become an incoherent mixture of three components: a τ component, a component $\ell_{\tilde{\tau}_2}^{\rm P}$ (the projection of the ℓ_2 component on the $\tilde{\tau}$ plane), and finally a component $\ell_{\tilde{\tau}_{\tilde{2}}}^{\rm P}$ (the projection of the ℓ_2 orthogonal component on the $\tilde{\tau}$ plane). The residual value of the pre-existing asymmetry can be then decomposed correspondingly as the sum of three terms,

$$N_{B-L}^{\rm p}(10^{12}\,{\rm GeV} \gg T \gg M_1) = N_{\Delta_\tau}^{\rm p} + N_{\Delta_{\tilde{\tau}_2}}^{\rm p} + N_{\Delta_{\tilde{\tau}_{\tilde{2}}}}^{\rm p} \,, \tag{49}$$

where

$$
\begin{aligned}
N_{\Delta_\tau}^{\rm p} &= [p_{2\tau}\, N_{\Delta_2}^{\rm p}(T_{B3})\, e^{-\frac{3\pi}{8} K_2} + p_{\tilde{2}\tau}\, N_{\Delta_{\tilde{2}}}^{\rm p}(T_{B3})] \tag{50}\\
N_{\Delta_{\tilde{\tau}_2}}^{\rm p} &= (1 - p_{2\tau})\, N_{\Delta_2}^{\rm p}(T_{B3})\, e^{-\frac{3\pi}{8} K_2}\\
N_{\Delta_{\tilde{\tau}_{\tilde{2}}}}^{\rm p} &= (1 - p_{\tilde{2}\tau})\, N_{\Delta_{\tilde{2}}}^{\rm p}(T_{B3})\,. \tag{51}
\end{aligned}
$$

When finally the N_1 wash-out processes act on the pre-existing asymmetry, one has to distinguish the wash-out acting on $N_{\Delta_\tau}^{\rm p}$, ruled by $K_{1\tau} \equiv p_{1\tau}\, K_1$, and the wash-out acting

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on $N^{\mathrm{P}}_{\Delta_{\tilde{\tau}_2}}$, ruled by $K_{1\tilde{\tau}} \equiv (1 - p_{1\tau}) K_1$. At the end of this stage, at $T \sim T_{B1}$, $N^{\mathrm{P}}_{\Delta_\tau}$ will be therefore given by

$$N^{\mathrm{P}}_{\Delta_\tau}(T_{B1}) = [p_{2\tau} N^p_{\Delta_2}(T_{B3}) e^{-\frac{3\pi}{8} K_2} + p_{\tilde{2}\tau} N^{\mathrm{P}}_{\Delta_{\tilde{2}}}(T_{B3})] e^{-\frac{3\pi}{8} K_{1\tau}}. \tag{52}$$

Imposing $K_{1\tau} \gtrsim 10$, one can enforce a strong wash-out of this component. At the same time, imposing $K_2 \gtrsim 10$, one can also enforce the wash-out of $N^{\mathrm{P}}_{\Delta_{\tilde{\tau}_2}}$. On the other hand the contribution $N_{\Delta^{\mathrm{P}}_{\tilde{2}}}$ does not undergo any N_2 wash-out. This part of the residual pre-existing asymmetry can be in turn decomposed as the sum of two terms: a term $N^{\mathrm{P}}_{\Delta_{\tilde{\tau}_1}}$, the asymmetry in the component of lepton states that are a projection of $\ell_{\tilde{\tau}_2}$ on $\ell_{\tilde{\tau}_1}$ and a term $N^{\mathrm{P}}_{\Delta_{\tilde{\tau}_{\tilde{1}}}}$, the asymmetry in the states orthogonal both to ℓ_1 and to ℓ_τ. The first one is exponentially washed-out by N_1 inverse processes,

$$N^{\mathrm{P}}_{\Delta_{\tilde{\tau}_1}}(T_{B1}) = p_{1\tilde{\tau}} (1 - p_{\tilde{2}\tau}) N^{\mathrm{P}}_{\Delta_{\tilde{2}}}(T_{B3}) e^{-\frac{3\pi}{8} (K_1 - K_{1\tau})}, \tag{53}$$

but the latter is not,

$$N^{\mathrm{P}}_{\Delta_{\tilde{\tau}_{\tilde{1}}}}(T_{B1}) = (1 - p_{\tilde{\tau}1}) (1 - p_{\tilde{2}\tau}) N^{\mathrm{P}}_{\Delta_{\tilde{2}}}(T_{B3}). \tag{54}$$

If we now go back to the expression eq. (36) for $N^{\mathrm{P}}_{\Delta_{\tilde{2}}}(T_{B3})$, we can see that even imposing $K_3 \gtrsim 10$ there will be still a completely unwashed term in the orthogonal ℓ_3 component, given by

$$N^{\mathrm{P}}_{\Delta_{\tilde{\tau}_{\tilde{1}}(\tilde{2}\tilde{3})}}(T_{B1}) = (1 - p_{\tilde{\tau}1}) (1 - p_{\tilde{2}\tau}) (1 - p_{\tilde{3}2}) (1 - p_{p3}) N^{\mathrm{p,i}}_{B-L}. \tag{55}$$

Therefore, in this scenario, even imposing $K_{1\tau}, K_2, K_3, (K_1 - K_{1\tau}) \gtrsim 10$, one has

$$w^{\mathrm{f}} \simeq (1 - p_{\tilde{\tau}1}) (1 - p_{\tilde{2}\tau}) (1 - p_{\tilde{3}2}) (1 - p_{p3}). \tag{56}$$

This shows that in general, except for some reduction due to a geometrical projection, there is no efficient wash-out of the pre-existing asymmetry. Therefore, it is clear that even in this scenario a sensible fraction of an arbitrary pre-existing asymmetry escapes the wash-out of all three RH neutrinos. The major stages of this scenario are summarized in Fig. 4.

4.1.2 Case $M_3 \gg 10^{12}\,\mathrm{GeV} \gg M_2 \gg M_1 \gg 10^9\,\mathrm{GeV}$

It is straightforward to extend the result of eq. (56) to a scenario where $M_3 \gg 10^{12}\,\mathrm{GeV} \gg M_2 \gg M_1 \gg 10^9\,\mathrm{GeV}$. Let us then just quickly discuss a derivation of the wash-out factor that is slightly different from the one in the previous case.

This time we have to impose $K_3 \gtrsim 10$ in order to wash-out first, at $T \sim M_3$, the component $N^{\mathrm{P}}_{\Delta_3} = p_{p3} N^{\mathrm{p,i}}_{B-L}$. At $T \sim 10^{12}\,\mathrm{GeV}$ the lepton quantum states become an

(a) $T \sim M_2$ (b) $T \sim 10^{12}\,\mathrm{GeV}$ (c) $T \sim M_1$

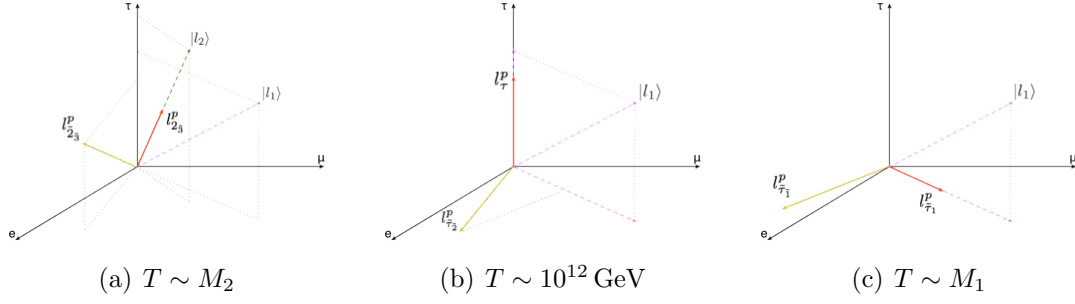Figure 4: The three most significant steps in the evolution of the residual pre-existing asymmetry in the scenario with $M_{2,3} \gg 10^{12}\,\mathrm{GeV}$ and $10^{12}\,\mathrm{GeV} \gg M_1 \gg 10^9\,\mathrm{GeV}$.

incoherent mixture of a τ component and of a $\tilde{\tau}$ component. A condition $K_{1\tau} + K_{2\tau} \gtrsim 10$ clearly guarantees a wash-out of the asymmetry in the τ component.

At $T \sim M_2$, the lepton quantum states $\tilde{\tau}$ component becomes a incoherent mixture of a component that is parallel to the ℓ_2 projection on the $\tilde{\tau}$ plane, that we indicate with $\ell_{\tilde{\tau}_2}$, and of a component $\ell_{\tilde{\tau}_{\bar{2}}}$, the projection of the ℓ_2 orthogonal component on the $\tilde{\tau}$ plane. A condition $K_{2\tilde{\tau}} = K_{2e} + K_{2\mu} \gtrsim 10$ guarantees the washout of the fraction of the pre-existing asymmetry in the first component but not of the fraction in the second one.

Finally, at $T \sim M_1$, the lepton quantum states become an incoherent mixture of a $\ell_{\tilde{\tau}_1}$ component and of a $\ell_{\tilde{\tau}_{\bar{1}}}$ component and again, imposing $K_{1\tilde{\tau}} = K_{1e} + K_{1\mu} \gtrsim 10$, one can enforce the wash-out of the asymmetry in the first component but not in the second one. Therefore, at the end there will still be a completely unwashed residual fraction of the pre-existing asymmetry given by

$$N_{B-L}^{\mathrm{p,f}} \simeq \left(1 - p_{\tilde{\tau}_2 \tilde{\tau}_1}\right) \left(1 - p_{\tilde{\tau}_3 \tilde{\tau}_2}\right) \left(1 - p_{\bar{3}\tau}\right) \left(1 - p_{p3}\right) N_{B-L}^{\mathrm{p,i}}, \tag{57}$$

showing again that the wash-out of $N_{B-L}^{\mathrm{p,i}}$ cannot be enforced in this scenario as well.

4.1.3   Case $10^{12}\,\mathrm{GeV} \gg M_3 \gg M_2 \gg M_1 \gg 10^9\,\mathrm{GeV}$

Finally, in the last scenario with $10^{12}\,\mathrm{GeV} \gg M_3 \gg M_2 \gg M_1 \gg 10^9\,\mathrm{GeV}$, the result for the final wash-out factor is a clear extension of the two previous cases and we can directly write the final result given by

$$w^{\mathrm{f}} \simeq \left(1 - p_{\tilde{\tau}_2 \tilde{\tau}_1}\right) \left(1 - p_{\tilde{\tau}_3 \tilde{\tau}_2}\right) \left(1 - p_{\tilde{\tau}_p \tilde{\tau}_3}\right) \left(1 - p_{p\tau}\right), \tag{58}$$

showing that in general, even in this case, $N_{B-L}^{\mathrm{p,i}}$ cannot be completely washed-out.

Therefore, we can conclude that in all mass patterns with $M_1 \gg 10^9\,\mathrm{GeV}$ it is not possible to enforce an efficient washout of a large pre-existing asymmetry.

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4.2 Scenarios with $M_1 \ll 10^9\,\mathrm{GeV}$

Let us now discuss those mass patterns where at least one $M_i \ll 10^9\,\mathrm{GeV}$. There are 6 different possibilities, as sketched in Fig. 5. It is quite clear, from our previous discussions,
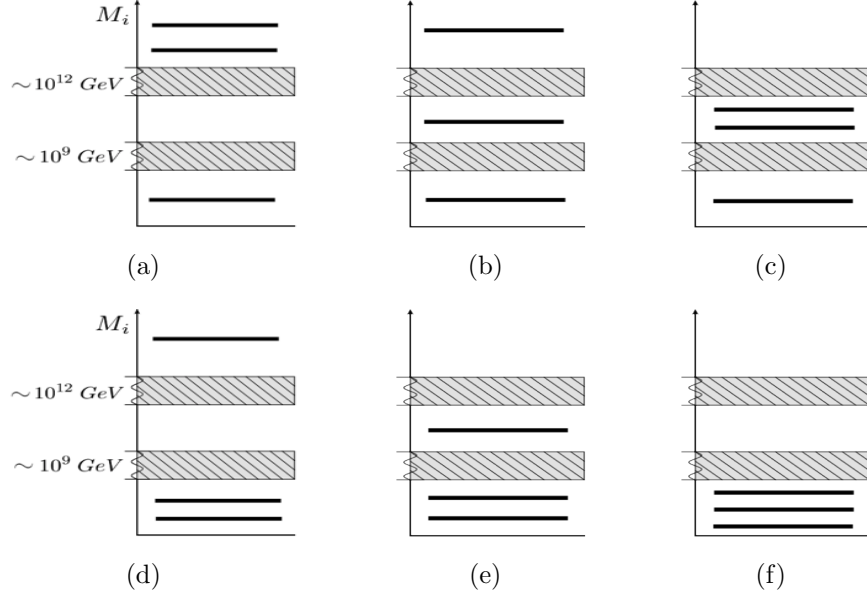

Figure 5: The six possible mass patterns with $M_1 \ll 10^9\,\mathrm{Gev}$. Only (b) and (c) allow for a successful strong thermal leptogenesis.

that for all them it is always possible to enforce a strong wash-out of the pre-existing asymmetry imposing $K_{1e}, K_{1\mu}, K_{1\tau} \gtrsim 10$ [15]. Indeed if at $T \sim T_{B2}$, when the N_2-inverse processes wash-out freeze, the residual value of the pre-existing asymmetry is given by $N^{\mathrm{p}}_{B-L}(T \sim T_{B2})$, then at $T \sim M_1 \ll 10^9\,\mathrm{GeV}$, irrespectively of the value of T_{B2}, this will be distributed in leptons and anti-leptons quantum states that are an incoherent mixture of the three light neutrino flavours. Therefore, the N_1 wash-out will act separately on each flavour contribution $N^{\mathrm{p}}_{\Delta_{e,\mu,\tau}}$ to the total residual pre-existing asymmetry $N^{\mathrm{p}}_{B-L}(T_{B2})$. In this way, at the end of the N_1 washout, the final value of the residual fraction of the pre-existing asymmetry is given by

$$N^{\mathrm{p,f}}_{B-L} = \sum_{\alpha=e,\mu,\tau} N^{\mathrm{p}}_{\Delta_\alpha}\, e^{-\frac{3\pi}{8}\,K_\alpha}\,. \tag{59}$$

Therefore, imposing $K_{1e}, K_{1\mu}, K_{1\tau} \gtrsim 10$, one can this time always enforce a sufficiently strong washout of a pre-existing large asymmetry $(w^{\mathrm{f}} \lesssim 10^{-8})$. However, such a strong condition would also wash-out the contribution $N^{\mathrm{lep,f}}_{B-L}$ produced from the two heaviest RH

neutrino decays while on the other hand, the lightest RH neutrino CP asymmetries are too low to have enough asymmetry produced from the N_1 decays [37]. Therefore, this condition is incompatible with successful leptogenesis [9].

We have then to find a weaker condition that can satisfy simultaneously $w^{\rm f} \lesssim 10^{-8}$ and $N_{B-L}^{\rm lep,f} \sim 10^{-7}$ or, using a vivid analogy, the way not to throw the baby out with the bath water. It is clear that at least one of the $K_{1\alpha}$ has to be $\lesssim 1$. The wash-out of the pre-existing asymmetry has then to be carried out by the heavier RH neutrinos and their decays have also to produce $N_{B-L}^{\rm lep,f}$ in the α flavour after the freeze-out of the wash-out processes.

4.2.1   Case $M_{2,3} \gg 10^{12}\,{\rm GeV}$

Let us start from a mass pattern where $M_{2,3} \gg 10^{12}\,{\rm GeV}$. At $T \sim T_{B2}$ the residual value of the pre-existing asymmetry is given by the eq. (41) and, imposing $K_2 \gtrsim 10$, only the asymmetry in the $\ell_{\bar 2}$ states survives (the second term in the eq. (41)). At $T \sim 10^{12}\,{\rm GeV}$, all lepton quantum states become an incoherent mixture of a τ component and of a $\tilde\tau$ component. The asymmetry produced from the N_2 decays at $T \sim T_{B2}$ is, by definition, all in the ℓ_2 flavour, i.e. $N_{B-L}^{\rm lep}(T_{B2}) = N_{\Delta_{\ell_2}}^{\rm lep}(T_{B2})$.

Below $T \sim 10^9\,{\rm GeV}$, both the two contributions to the asymmetry in states in the $\tilde\tau$ plane, the residual pre-existing fraction and the one produced by RH neutrino decays, will be distributed in quantum lepton states that are an incoherent mixture of a muon and of an electron component. Therefore, there is a residual fraction of the pre-existing asymmetry in each light lepton flavour. This implies that it is impossible to impose a condition such that all the residual pre-existing asymmetry is washed-out without also washing out the contribution produced from RH neutrino decays.

4.2.2   Case $10^{12}\,{\rm GeV} \gg M_2 \gg 10^9\,{\rm GeV}$

Finally, we consider a scenario with $M_1 \ll 10^9\,{\rm GeV}$ and $10^{12}\,{\rm GeV} \gg M_2 \gg 10^9\,{\rm GeV}$. At $T \sim M_2$, the lepton quantum states become an incoherent mixture of a τ component and of a $\tilde\tau$ component. We can again impose $K_{2\tau} \gtrsim 10$ in such a way that any residual pre-existing asymmetry $\lesssim \mathcal{O}(1)$ in the tauon flavour is washed-out. However, this time the out-of-equilibrium N_2-decays can still afterwards produce an asymmetry in the same

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[9]There is a loophole. In [38] it was shown that the flavoured CP asymmetries contain a term that is not upper bounded if one allows for strong cancellations in the light neutrino masses from the seesaw formula corresponding to large entries of the orthogonal matrix. Successful leptogenesis from N_1 decays is then possible for $M_1 \ll 10^9\,{\rm GeV}$.

tauon flavour sufficient to have successful leptogenesis. The asymmetry has necessarily to be produced in the tauon flavour since we have still to enforce a strong wash-out of the pre-existing asymmetry in the electron and muon flavours from the N_1 inverse processes by imposing $K_{1e}, K_{1\mu} \gg 1$. Notice that successful leptogenesis requires $\varepsilon_{2\tau} \sim 10^{-6}$, so that at the end $\eta_B \sim \varepsilon_{2\tau}\, \kappa(K_{2\tau}) \sim 10^{-9}$. In this way we have finally pinned down a scenario that can realize successful strong thermal leptogenesis: *a tauon N_2-dominated scenario* [10]. It is interesting to notice that in this scenario the presence of a third RH neutrino specie N_3 is necessary for $\varepsilon_{2\tau}$ not to be suppressed as $\sim (M_1/M_2)\, 10^{-6}\, M_1/10^{10}\, \mathrm{GeV}$.

It is easy to understand that there cannot be a scenario of successful strong thermal leptogenesis where the final asymmetry is dominantly in the electron or in the muon flavour. Suppose that we impose $K_{2e} + K_{2\mu} \gtrsim 10$, so that all the pre-existing asymmetry in the $\tilde{\tau}_2$ component is washed out at $T \sim M_2$. Suppose that afterwards a sufficiently high asymmetry is generated in the $\tilde{\tau}_2$ component by out-of-equilibrium N_2 decays at $T \sim T_{B2}$. However, there is still a $\tilde{\tau}_{\bar{2}}$ component $N^p_{\Delta_{\tilde{\tau}_{\bar{2}}}}(T_{B2})$, that escapes the wash-out.

Indeed at $T \ll 10^9\, \mathrm{GeV}$, the lepton quantum states become an incoherent mixture of an electron and of a muon component. If we impose $K_{1\tau} + K_{2\tau} \gtrsim 10$ we can wash-out efficiently the residual pre-existing asymmetry in the tauon flavour. However, either K_{1e} or $K_{1\mu}$, has to be necessarily $\lesssim 1$ otherwise everything, including N^{lep}_{B-L}, would be washed-out. For definiteness let us choose $K_{1e} \lesssim 1$. This implies that there will be still a final residual value of the asymmetry in the electron flavour given by

$$N^{f,p}_{\Delta_e} = p_{\tilde{\tau}_{\bar{2}}e}\, N^p_{\Delta_{\tilde{\tau}_{\bar{2}}}}(T_{B2}) \tag{60}$$

that cannot be washed-out. Of course the same would happen if we would impose $K_{1\mu} \lesssim 1$ instead of $K_{1e} \lesssim 1$.

Notice that if we consider a mass pattern where both M_2 and M_3 are in the range $[10^9, 10^{12}]\, \mathrm{GeV}$, then things would work exactly in the same way. Simply in this case one can impose the less restrictive condition $K_{2\tau} + K_{3\tau} \gtrsim 10$ in order to wash-out the pre-existing asymmetry in the tauon flavour. Notice that a tauon N_3-dominated scenario could be in principle possible if $K_{3\tau} \gtrsim 10$ and $K_{2\tau} \lesssim 1$. However, the maximal value of $\varepsilon_{3\tau}$ is suppressed as $\propto M_2/M_3$ with respect to $\varepsilon_{2\tau}$ and therefore the asymmetry produced from N_2 decays tends to be much higher both because of the lower wash-out and because of the much higher *CP* asymmetry. Therefore, this possibility can be possibly realized

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[10]Notice that in a tauon N_2-dominated scenario with $K_{2\tau} \gg 1$ (strong wash-out regime) one has an expression for the final asymmetry that is basically given by the expression valid in the traditional unflavoured N_1 dominated scenario where K_1 is replaced by $K_{2\tau}$. In this case one obtains (see for example right panel of Fig. 1 in [14]) an upper bound $K_{2\tau} \ll 1000$ from the condition $M_2 \ll 10^{12}\, \mathrm{GeV}$

(a) $T \sim T_{B3}$

(b) $T \sim 10^{12}\,\mathrm{GeV}$
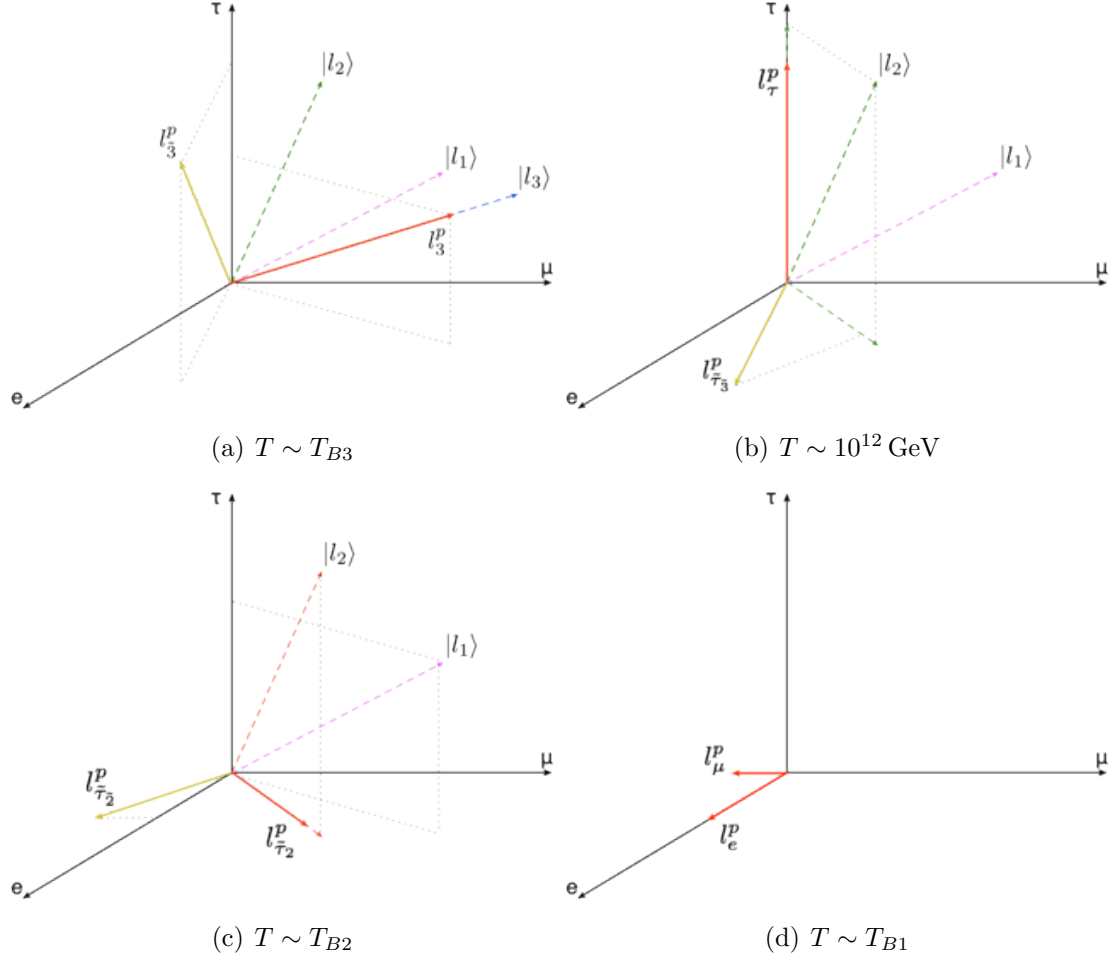
(c) $T \sim T_{B2}$

(d) $T \sim T_{B1}$

Figure 6: Tauon N_2 dominated scenario. Steps in the evolution of the lepton quantum states carrying the residual pre-existing asymmetry. As in the previous figures, the red components are those undergoing the wash-out, while the yellow component escapes the wash-out. The yellow component, tracking the lepton states carrying an unwashed fraction of the pre-existing asymmetry, is not present after the N_1 wash-out at $T \sim M_1$. We have finally singled out the only mass pattern where successful strong thermal leptogenesis is possible.

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only for a very fine tuned choice of the parameters and in any case not for a too strong hierarchy between M_2 and M_3. For this reason mass patterns with $M_2 \ll 10^9\,\text{GeV}$ and $M_3 \gg 10^9\,\text{GeV}$ cannot give successful leptogenesis.

In this way we have finally shown that the only possible scenario with a hierarchical RH neutrino mass spectrum which allows for successful strong thermal leptogenesis is a tauon N_2-dominated scenario, where $M_1 \ll 10^9\,\text{GeV}$ and $10^{12}\,\text{GeV} \gg M_2 \gg 10^9\,\text{GeV}$ and the final asymmetry is dominantly in the tauon flavour. In Fig. 6 we have sketched the 4 main steps in the evolution of the flavour composition of N_{B-L}^{p}.

5   Remarks, Caveats and Conclusions

Let us make some remarks and indicate some caveats about the results that we have found, in particular pointing out the limits of validity and some alternative ways to realize successful strong thermal leptogenesis relaxing some of the assumptions made so far.

5.1   Beyond the hierarchical limit

Our conclusions have been derived within the hierarchical limit. If even just the two lightest RH neutrinos masses are allowed to be almost-degenerate and lower than $10^9\,\text{GeV}$, then the CP asymmetries are enhanced [39] and successful leptogenesis is possible. At the same time, the asymmetry from the RH neutrino decays is produced in the three flavour regime. Therefore, before the onset of leptogenesis, a pre-existing asymmetry gets distributed into quantum lepton states that are an incoherent mixture of all three light flavour eigenstates. In this case, as already discussed, an efficient wash-out can be enforced imposing $K_{1\alpha} \gtrsim 10$ for all α without spoiling successful leptogenesis. Therefore, this is a clear loophole in our conclusions. This agrees with previous results obtained within the context of resonant tau-leptogenesis [40].

Less trivially, it should be noticed that a scenario where all three RH neutrino masses are quasi-degenerate should realize successful strong thermal leptogenesis for any value of M_i (e.g. even in the case of the heavy flavoured scenario of Fig. 1). Our results were obtained under the assumption that the wash-out from different RH neutrinos inverse processes occur at different stages. In this way the wash-out of the asymmetry occurs first along one direction, e.g. ℓ_3, and then along a different one, e.g. ℓ_2. This implies that at any stage there is a well defined flavour basis where the density matrix is diagonal. If the wash-out occurs simultaneously along three different independent directions, even not orthogonal to each other, then a pre-existing asymmetry should be efficiently washed-out

anyway. However, this heuristic argument would require to be more rigorously proved within a density matrix formalism.

5.2   Supersymmetric models

In the case of supersymmetric models, charged lepton interactions can become considerably faster and the condition eq. (13) for the full projection of the leptonic quantum states on the light flavor basis gets relaxed to [41]

$$M_i \ll 10^9 \, \mathrm{GeV} \, (1 + \tan^2 \beta). \tag{61}$$

On the other hand, the lower bound on the RH neutrino mass for successful leptogenesis in the strong wash-out regime, $M_1 \gtrsim 2 \times 10^{10} \, \mathrm{GeV} \, (K_{1\alpha}/10)^{1.2}$, does not change significantly [42, 14]. Therefore, for $1 + \tan^2 \beta \gtrsim 20$, it is possible to have a full wash-out of the pre-existing asymmetry imposing $K_{1e}, K_{1\mu}, K_{1\tau} \gtrsim 10$ and at the same time successful leptogenesis from light RH neutrino decays. In this way a traditional N_1-dominated leptogenesis scenario where the asymmetry is produced in the three flavour regime is possible. Of course it is also still possible to realize successful strong thermal leptogenesis within a N_2-dominated scenario, for $K_{2\alpha} \gtrsim 10$ and $K_{1\alpha} \lesssim 1$, this time in an arbitrary flavour α.

5.3   Light neutrino flavour coupling

Light flavour asymmetries do not evolve independently of each other, and in a N_2-dominated scenario, light flavour coupling can play a relevant role [27]. In our case the main issue is whether flavour coupling can spoil the efficient wash-out of the pre-existing asymmetry in the tauon N_2-dominated scenario allowing the electron and muon flavour asymmetries to escape the wash-out in general. This is because the eq. (59) for the final total asymmetry has now to be replaced by a much more general equation for the final individual flavour contributions to the asymmetry

$$N_{\Delta_\alpha}^{\mathrm{f}} \;\; = \;\; \sum_{\alpha''} V_{\alpha\alpha''}^{-1} \left[N_{\alpha''}^{T \sim T_{B2}} \, e^{-\frac{3\pi}{8} K_{1\alpha''}} \right] ,$$

where $V_{\alpha\alpha''}$ is a mixing matrix that can be derived from the flavour coupling mixing matrix and the $N_{\alpha''}^{T \sim T_{B2}}$'s are a linear combination of the $N_{\alpha}^{T \sim T_{B2}}$'s. In first approximation the matrix V is diagonal and one recovers the eq. (59) but in general one can see how the wash-out for example of the the μ contribution can partly proceed through an exponential containing $K_{1\tau}$. Therefore, in principle, this component can escape the wash-out in the

tauon N_2-dominated scenario. However, as one can see, the result depends in a complicated way on the seesaw parameters and it should be checked at each point in the seesaw parameter space whether a pre-existing asymmetry is efficiently washed out or not. Light neutrino flavour coupling could therefore potentially introduce further restrictions on the tauon N_2-dominated scenario.

5.4   Heavy neutrino flavour coupling

As for the light neutrino flavour asymmetries, the dynamics of the heavy neutrino flavour asymmetries also couple when spectator processes are considered [15]. For example, in the heavy flavoured scenario, for $K_3 \gtrsim 10$, one would have that the $N_{\Delta_3} = -N_{\Delta_{\bar{3}}}/4$, implying that part of the component of pre-existing asymmetry in the ℓ_3 quantum states is not completely washed out. This result can be understood considering that when the Higgs asymmetry N_H is taken into account in the kinetic equation for N_{Δ_3} (cf. eq. (39)),

$$\frac{dN_{\Delta_3}^{\mathrm{p}}}{dz_3} = -W_3 \left(N_{\Delta_3}^{\mathrm{p}} + \frac{N_H}{2} \right) , \tag{62}$$

it couples the wash-out of $N_{\Delta_3}^{\mathrm{p}}$ to the value of $N_{\Delta_{\bar{3}}}^{\mathrm{p}}$. At the same time the Higgs asymmetry is enforced, by equilibrium of gauge and top Yukawa interactions plus baryon number and hypercharge conservation, to be $N_H = N_{\Delta_{\bar{3}}}/2$. However, an account of this effect would just introduce $\mathcal{O}(1)$ corrections to the wash-out factors without changing our conclusions.

5.5   An improved description of decoherence

As we have seen decoherence of lepton quantum states originates from two different interactions: the N_i inverse processes that tend to collapse the quantum states into a incoherent mixture of a ℓ_i component and of a $\ell_{\bar{i}}$ component and from charged lepton interactions that, for example, at $T \sim 10^{12} \, \mathrm{GeV}$ tend to collapse the lepton quantum states into a τ component and into a $\tilde{\tau}$ component. In the first case a description of decoherence in terms of density matrix equations cannot change our conclusions. It can only just slightly affect the precise value of $K_{\star}$, i.e. the minimum value for the K_i's and the $K_{i\alpha}$'s to get an efficient wash-out. On the other hand, in the second case it would be important to have an exact description in order to get the precise conditions on the M_i's in order for the different mass patterns to hold, in particular those ones for the N_2 dominated scenario. These conditions have been worked out in the calculation of $N_{B-L}^{\mathrm{lep,f}}$ [22] but they could be slightly different in the calculation of $N_{B-L}^{\mathrm{p,f}}$.

5.6 Phantom terms and dependence on the initial RH neutrino abundance

We have so far assumed that the flavour compositions of the pre-existing leptons and anti-leptons (cf. eqs. (9) and (10)) are the same. If we relax this assumption, we have to take into account the so called phantom terms [27] that give an additional way to a pre-existing asymmetry to avoid the wash-out in all scenarios considered but not in the tauon N_2-dominated scenario. The phantom terms give a contribution to the flavoured asymmetries coming from possible differences in the flavor composition of the pre-existing lepton and anti-lepton quantum states that do not vanish when the total number of leptons and of anti-leptons are the same.

The flavour composition of leptons and anti-leptons are respectively given by the eq. (9) and by the eq. (10). We have so far assumed that $\mathcal{C}_{p\alpha} = \bar{\mathcal{C}}_{p\alpha}$. Let us now drop this assumption. Let us first give as an example what happens at the end of the first stage, at temperatures $T \sim M_3$. The pre-existing flavour asymmetries can be again decomposed as in the eq. (26) but this time the terms $N_{\Delta_3}^{\mathrm{p,i}}$ and $N_{\Delta_{\tilde{3}}}^{\mathrm{p,i}}$ are the sum of two terms,

$$N_{\Delta_3}^{\mathrm{p,i}} = \frac{p_{p3} + \bar{p}_{p3}}{2}\, N_{B-L}^{\mathrm{p,i}} + \frac{p_{p3} - \bar{p}_{p3}}{2}\,(N_{\ell^{\mathrm{p}}} + N_{\bar{\ell}^{\mathrm{p}}}). \tag{63}$$

and

$$N_{\Delta_{\tilde{3}}}^{\mathrm{p,i}} = \frac{p_{p\tilde{3}} + \bar{p}_{p\tilde{3}}}{2}\, N_{B-L}^{\mathrm{p,i}} + \frac{\bar{p}_{p\tilde{3}} - \bar{p}_{p\tilde{3}}}{2}\,(N_{\ell^{\mathrm{p}}} + N_{\bar{\ell}^{\mathrm{p}}})\,. \tag{64}$$

The first terms are the usual contributions coming from a difference in the number of leptons and antileptons, the $N_{B-L}^{\mathrm{p,i}}$ asymmetry, that gets shared between the two flavours just proportionally to the average of the two probabilities that the lepton is in one of the two flavour. The second terms are the so called phantom terms [27] and are due to a different flavour composition of the lepton and anti-lepton quantum states. They are proportional to the number of leptons (in thermal equilibrium one has simply $N_{\ell^{\mathrm{p}}} \simeq N_{\bar{\ell}^{\mathrm{p}}} \simeq 1$). Notice that the sum of the two phantom terms vanishes at $T \sim M_3$ since $p_{p3} + p_{p\tilde{3}} = \bar{p}_{p3} + \bar{p}_{p\tilde{3}} = 1$. The phantom terms are such that when the associated flavour components (in the example ℓ_3 and $\ell_{\tilde{3}}$) are in a coherent superposition, then they cannot be washed-out. This is because the wash-out is just a statistical damping of the difference between the number of lepton and of anti-leptons of the incoherent components. This means that these terms escape the wash-out exactly like the orthogonal components but with the difference that their sum vanish. However, in a later stage, when the phantom terms flavour components are also measured, they can be finally washed out. If the washout proceeds in a asymmetric way, their sum does not vanish any more and the phantom terms would finally show up.

Let us give a second example that well illustrates this last statement. Consider the mass pattern in the left panel of Fig. 3. At the decoherence at $T \sim 10^{12}\,\mathrm{GeV}$, if phantom terms are neglected, the N_{Δ_τ} asymmetry is given by the first term in the RH side of eq. (49). If phantom terms are taken into account there would be an additional contribution given by

$$N_{\Delta_\tau}^{\mathrm{p,phantom}} = \frac{p_{p\tau} - \bar{p}_{p\tilde{\tau}}}{2} \left(N_{\ell_2^{\mathrm{p}}} + N_{\bar{\ell}_2^{\mathrm{p}}} \right), \tag{65}$$

and an opposite term $N_{\Delta_{\tilde{\tau}}}^{\mathrm{p,phantom}}$. Notice again that phantom terms are proportional to the total number of pre-existing leptons. At the lightest RH neutrino wash-out stage, at $T \sim M_1$, if for example $K_{1\tau} \lesssim 1$ and $K_{1\tilde{\tau}} \gtrsim 10$, then all the asymmetry $N_{\Delta_{\tilde{\tau}}}^{\mathrm{p,phantom}}$ is efficiently washed-out but the asymmetry $N_{\Delta_\tau}^{\mathrm{p,phantom}}$ survives.

Analogously one would have phantom terms in the electron and in the muon flavour in the N_2 dominated scenario after the decoherence stage at $T \sim 10^9\,\mathrm{GeV}$.

It is however clear that an account of the phantom terms cannot spoil our conclusion about the tauon N_2 dominated scenario, i.e. they are also efficiently washed out. This is because when the electron and muon components become an incoherent mixture at $T \sim 10^9\,\mathrm{GeV}$, even though now one can have phantom terms that survived the previous N_2 wash-out along $\ell_{\tilde{\tau}_2}$, these are anyway efficiently washed out, together with the component along $\ell_{\tilde{\tau}_2}$, by the N_1 wash-out if $K_{1e}, K_{1\mu} \gtrsim 10$.

There is another aspect concerning the role of phantom terms. There are also phantom terms directly in the asymmetries generated by the RH neutrinos, due to a different flavour composition of the quantum states $|\ell_i\rangle$ with respect to the flavour composition of the CP conjugated $|\bar{\ell}_i'\rangle$ [27]. These phantom terms introduce an additional source of dependence on the initial abundance of RH neutrinos. However, again, their presence can only worsen the dependence on the initial conditions for all scenarios but the tauon N_2 dominated scenario. This is because also these phantom terms are efficiently washed out by the N_1 inverse processes if $K_{1e}, K_{1\mu} \gtrsim 10$. Therefore, if $K_{2\tau} \gtrsim K_\star$ [14], we can conclude that the tauon N_2-dominated scenario is fully independent of the initial conditions, both of an initial pre-existing asymmetry and of the initial RH neutrino abundance.

5.7   On the leptogenesis conspiracy

Notice that the usual observation that strong thermal leptogenesis is possible because the solar and the atmospheric neutrino mass scales are found to be just about one order of magnitude bigger than the equilibrium neutrino mass $m_\star$, typically implying $K_i = 10-50$, still holds now that flavour effects are taken into account. It is indeed still a necessary condition since otherwise it would not be possible to impose $K_{2\tau} + K_{3\tau}, K_{1e}, K_{1\mu} \gtrsim 10$

without fine tuning. Simply, as we have seen, when flavour effects are taken into account, additional conditions have to be imposed.

5.8 On the relevance of the requirement of strong thermal leptogenesis

One could legitimately just ignore the problem of the initial conditions, for example assuming conservatively vanishing RH neutrino abundances and vanishing pre-existing asymmetry. However, there are many reasons to think that at the end of inflation, or in any case before leptogenesis, the Universe was already in a CP non invariant state [9, 10, 11]. It would be therefore quite important, in this perspective, to investigate further these possible mechanisms of production of a a pre-existing asymmetry before the onset of leptogenesis. If it could be possible to conclude that a large $\mathcal{O}(1)$ asymmetry unavoidably emerges after inflation, then strong thermal leptogenesis would become a necessary unescapable requirement. However, even though at the moment we lack such a stringent motivation, any model that yields solutions respecting the strong thermal leptogenesis requirement should be regarded certainly more attractive, and, on more practical grounds, any kind of prediction that can be possible derived from it as more robust.

5.9 Conclusions

We have seen how a full account of heavy and light neutrino flavour effects gives many ways to a pre-existing asymmetry to escape the wash-out of the RH neutrinos. On the other hand, it is quite interesting that there is a well defined scenario, the tauon N_2 dominated scenario, where successful strong thermal leptogenesis is possible. If, to a first sight, flavour effects can therefore seem to spoil the attractiveness of thermal leptogenesis, it is also true that thanks to flavour effects the observed asymmetry can be successfully produced by the next-to-lightest RH neutrinos in quite a natural way, so that the lower bound on M_1 is basically nullified. In this way, models emerging from grand-unified theories, such as so called '$SO(10)$-inspired models' [4, 5], can well explain the observed asymmetry. It is then quite intriguing that in these models the final asymmetry is dominantly produced just in the tauon flavour and that all conditions for successful strong thermal leptogenesis that we have derived in this paper are fulfilled. It is also greatly interesting that these models seem to give rise to predictions on the low energy neutrino parameters that can be tested in future experiments during next years, opening new hopes to a testability of the minimal seesaw mechanism in combination with leptogenesis. In conclusion, from our

analysis, the tauon N_2 dominated scenario emerges as a particularly attractive realization of leptogenesis.

Appendix

Heavy Flavoured scenario

The heavy flavour basis

Given the Dirac mass matrix, the probabilities p_{ij}'s can be expressed as

$$p_{ij} = \frac{\left|(m_D^\dagger m_D)_{ij}\right|^2}{(m_D^\dagger m_D)_{ii}\,(m_D^\dagger m_D)_{jj}}\,. \tag{A.1}$$

In a basis where the charged lepton and the Majorana mass matrices are diagonal, the Dirac mass matrix can be parameterized in terms of the leptonic mixing matrix U and of the orthogonal matrix Ω [32] as $m_D = U\,\sqrt{D_m}\,\Omega\,\sqrt{D_M}$. From the unitarity of U, it easily follows that

$$p_{ij} = \frac{|\sum_h m_h\,\Omega_{hi}^\star\,\Omega_{hj}|^2}{\widetilde{m}_i\,\widetilde{m}_j}\,. \tag{A.2}$$

In general $p_{ij} \neq \delta_{ij}$ (i.e. in general the heavy flavoured basis is not orthonormal) but for Ω equal to the unity matrix or to the five special forms obtained from the unity matrix permuting rows and columns it can be indeed verified, that $p_{ij} = \delta_{ij}$. These special forms correspond to so called form dominance models [33]. However, for these 6 special cases the total CP asymmetries vanish and successful leptogenesis is not attained [16] since RH neutrinos do not interfere $((m_D^\dagger m_D)_{i \neq j} = 0)$.

Wash-out factor

A full expression for the residual value of the pre-existing asymmetry in the heavy flavoured scenario can be written as the sum of three terms

$$N_{B-L}^{\mathrm{p,f}} = N_{\Delta_1}^{\mathrm{p,f}}(T_{B1}) + N_{\Delta_{\tilde{1}_2}}^{\mathrm{p,f}}(T_{B1}) + N_{\Delta_{\tilde{1}_{\bar{2}}}}^{\mathrm{p,f}}(T_{B1})\,. \tag{A.3}$$

The first term is the contribution to the residual value of the pre-existing asymmetry in ℓ_1 lepton quantum states and is given by

$$
\begin{aligned}
N_{\Delta_1}^{\mathrm{p,f}}(T_{B1}) \;=\; & N_{B-L}^{\mathrm{p,i}} \left[p_{21}\, p_{32}\, p_{p3}\, e^{-\frac{3\pi}{8}(K_1+K_2+K_3)} \right. \\
& + p_{21}\, p_{\bar{3}2}\, (1-p_{p3})\, e^{-\frac{3\pi}{8}(K_1+K_2)} \\
& + p_{\bar{2}_3 1}\, (1-p_{32})\, p_{p3}\, e^{-\frac{3\pi}{8}(K_1+K_3)} \\
& \left. + p_{\bar{2}_{\bar{3}} 1}\, (1-p_{\bar{3}2})\, (1-p_{p3})\, e^{-\frac{3\pi}{8} K_1} \right].
\end{aligned}
\tag{A.4}
$$

The second term is the contribution in $\ell_{\tilde{1}_2}$ lepton quantum states given by

$$
\begin{aligned}
N_{\Delta_{\tilde{1}_2}}^{\mathrm{p,f}}(T_{B1}) \;=\; & N_{B-L}^{\mathrm{p,i}} \left[(1-p_{21})\, p_{32}\, p_{p3}\, e^{-\frac{3\pi}{8}(K_2+K_3)} \right. \\
& \left. + (1-p_{21})\, p_{\bar{3}2}\, (1-p_{p3})\, e^{-\frac{3\pi}{8} K_2} \right].
\end{aligned}
\tag{A.5}
$$

Finally, the third term is the contribution in $\ell_{\tilde{1}_{\hat{2}}}$ lepton quantum states given by

$$
\begin{aligned}
N_{\Delta_{\tilde{1}_{\hat{2}}}}^{\mathrm{p,f}}(T_{B1}) \;=\; & N_{B-L}^{\mathrm{p,i}} \left[(1-p_{\bar{2}1})\, (1-p_{32})\, p_{p3}\, e^{-\frac{3\pi}{8} K_3} \right. \\
& \left. + (1-p_{\bar{2}_{\bar{3}} 1})\, (1-p_{\bar{3}2})\, (1-p_{p3}) \right].
\end{aligned}
\tag{A.6}
$$

Imposing $K_1, K_2, K_3 \gg 1$ one can wash-out all terms except the last one, obtaining for the wash-out factor the expression (44). It can be checked that the sum of all 8 probabilities adds to unity.

Asymmetry from leptogenesis

Let us give the expression for N_{B-L}^{lep} in the three different stages of the heavy flavoured scenario (it is the most involved case, extension to other scenarios is straightforward) assuming $T_i \gg M_3$ and neglecting heavy flavour coupling [15].

At the end of the N_3 wash-out stage the asymmetry produced from N_3 decays in the ℓ_3 states (from the eqs. (39)) is simply given by

$$
N_{B-L}^{\mathrm{lep}}(T_{B3}) = N_{\Delta_3}^{\mathrm{lep}}(T_{B3}) = \varepsilon_3\, \kappa(K_3).
\tag{A.7}
$$

At the end of the N_2 wash-out stage, the asymmetry can be written as the sum of two components,

$$
N_{B-L}^{\mathrm{lep}}(T_{B2}) = N_{\Delta_2}^{\mathrm{lep}}(T_{B2}) + N_{\Delta_{\hat{2}_3}}^{\mathrm{lep}}(T_{B2}).
\tag{A.8}
$$

The first components is the asymmetry in ℓ_2 states. This is washed out by N_2 inverse processes and is in turn the sum of two terms,

$$
N_{\Delta_2}^{\mathrm{lep}}(T_{B2}) = p_{32}\, \varepsilon_3\, \kappa(K_3)\, e^{-\frac{3\pi}{8} K_2} + \varepsilon_2\, \kappa(K_2),
\tag{A.9}
$$

the first one is the asymmetry produced by N_3 decays while the second one is the asymmetry produced by N_2 decays.

The second term in the eq. (A.8), in $\ell_{\bar{2}_3}$ quantum states, is the asymmetry produced by N_3 decays that is not washed-out by N_2 inverse processes and can be written as

$$N^{\rm lep}_{\Delta_{\bar{2}_3}}(T_{B2}) = (1 - p_{32})\,\varepsilon_3\,\kappa(K_3)\,. \tag{A.10}$$

At the end of the N_1 wash-out we can again write the total asymmetry as the sum of two terms

$$N^{\rm lep}_{B-L}(T_{B1}) = N^{\rm lep}_{\Delta_1}(T_{B1}) + N^{\rm lep}_{\Delta_{\bar{1}}}(T_{B1})\,, \tag{A.11}$$

where

$$
\begin{aligned}
N^{\rm lep}_{\Delta_1}(T_{B1}) \;=\;& p_{21}\,p_{32}\,\varepsilon_3\,\kappa(K_3)\,e^{-\frac{3\pi}{8}\,(K_1+K_2)} \\
&+ p_{21}\,\varepsilon_2\,\kappa(K_2)\,e^{-\frac{3\pi}{8}\,K_1} \\
&+ p_{\bar{2}_31}\,(1 - p_{32})\,\varepsilon_3\,\kappa(K_3)\,e^{-\frac{3\pi}{8}\,K_1} \\
&+ \varepsilon_1\,\kappa(K_1)
\end{aligned}
\tag{A.12}
$$

and

$$
\begin{aligned}
N^{\rm lep}_{\Delta_{\bar{1}}}(T_{B1}) \;=\;& (1 - p_{21})\,[\,p_{32}\,\varepsilon_3\,\kappa(K_3)\,e^{-\frac{3\pi}{8}\,K_2} + \varepsilon_2\,\kappa(K_2)\,] \\
&+ (1 - p_{\bar{2}_31})\,(1 - p_{32})\,\varepsilon_3\,\kappa(K_3)\,.
\end{aligned}
\tag{A.13}
$$

This calculation can be easily extended to the asymmetry produced from RH neutrino decays in the other mass patterns.

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